DAKOTA, A Multilevel Parallel Object-Oriented Framework for
Design Optimization, Parameter Estimation, Uncertainty
Quantification, and Sensitivity Analysis

Version 5.1 Reference Manual

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Abstract

The DAKOTA (Design Analysis Kit for Optimization and Terascale Applications) toolkit provides a flexible and extensible interface between simulation codes and iterative analysis methods. DAKOTA contains algorithms for optimization with gradient and nongradient-based methods; uncertainty quantification with sampling, reliability, and stochastic finite element methods; parameter estimation with nonlinear least squares methods; and sensitivity/variance analysis with design of experiments and parameter study methods. These capabilities may be used on their own or as components within advanced strategies such as surrogate-based optimization, mixed integer nonlinear programming, or optimization under uncertainty. By employing object-oriented design to implement abstractions of the key components required for iterative systems analyses, the DAKOTA toolkit provides a flexible and extensible problem-solving environment for design and performance analysis of computational models on high performance computers.

This report serves as a reference manual for the commands specification for the DAKOTA software, providing input overviews, option descriptions, and example specifications.
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Chapter 1

DAKOTA Reference Manual

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1.1 Overview

The DAKOTA (Design Analysis Kit for Optimization and Terascale Applications) toolkit provides a flexible, extensible interface between analysis codes and iteration methods. DAKOTA contains algorithms for optimization with gradient and nongradient-based methods, uncertainty quantification with sampling, reliability, stochastic expansion, and interval estimation methods, parameter estimation with nonlinear least squares methods, and sensitivity/variance analysis with design of experiments and parameter study capabilities. (Solution verification and Bayesian approaches are also in development.) These capabilities may be used on their own or as components within advanced algorithms such as surrogate-based optimization, mixed integer nonlinear programming, mixed aleatory-epistemic uncertainty quantification, or optimization under uncertainty. By employing object-oriented design to implement abstractions of the key components required for iterative systems analyses, the DAKOTA toolkit provides a flexible problem-solving environment as well as a platform for rapid prototyping of new solution approaches.

The Reference Manual focuses on documentation of the various input commands for DAKOTA. It follows closely the structure of dakota.input.summary, the master input specification summary. For information on software structure, refer to the Developers Manual, and for a tour of DAKOTA features and capabilities, refer to the Users Manual [Adams et al., 2010].

1.2 Input Specification Reference

In DAKOTA, the strategy creates and manages iterators and models. A model contains a set of variables, an interface, and a set of responses, and the iterator operates on the model to map the variables into responses using the interface. In a DAKOTA input file, the user specifies these components through strategy, method, model, variables, interface, and responses keyword specifications. The Reference Manual closely follows this structure, with introductory material followed by detailed documentation of the strategy, method, model, variables, interface,
and responses keyword specifications:

Introduction
Strategy Commands
Method Commands
Model Commands
Variables Commands
Interface Commands
Responses Commands

1.3 Additional Resources

A bibliography for the Reference Manual is provided in:

Bibliography

2.1 dakota.input.summary File Reference

File containing the input specification for DAKOTA.

2.1.1 Detailed Description

File containing the input specification for DAKOTA.

This file is derived automatically from dakota.input.nspec, which is used in the generation of parser system files that are compiled into the DAKOTA executable. Therefore, these files are the definitive source for input syntax, capability options, and associated data inputs. Refer to Instructions for Modifying DAKOTA’s Input Specification for information on how to modify the input specification and propagate the changes through the parsing system.

Key features of the input specification and the associated user input files include:

- In the input specification, required individual specifications simply appear, optional individual and group specifications are enclosed in [], required group specifications are enclosed in (), and either-or relationships are denoted by the | symbol. These symbols only appear in dakota.input.nspec; they must not appear in actual user input files.

- Keyword specifications (i.e., strategy, method, model, variables, interface, and responses) begin with the keyword possibly preceded by white space (blanks, tabs, and newlines) both in the input specifications and in user input files. For readability, keyword specifications may be spread across several lines. Earlier versions of DAKOTA (prior to 4.1) required a backslash character (\) at the ends of intermediate lines of a keyword. While such backslashes are still accepted, they are no longer required.

- Some of the keyword components within the input specification indicate that the user must supply INTEGER, REAL, STRING, INTEGERLIST, REALLIST, or STRINGLIST data as part of the specification. In a user input file, the "=" is optional, data in a LIST can be separated by commas or whitespace, and the STRING data are enclosed in single or double quotes (e.g., ‘text_book’ or "text_book").
In user input files, input is largely order-independent (except for entries in lists of data), case insensitive, and white-space insensitive. Although the order of input shown in the Sample dakota.in Files generally follows the order of options in the input specification, this is not required.

In user input files, specifications may be abbreviated so long as the abbreviation is unique. For example, the npsol_sqp specification within the method keyword could be abbreviated as npsol, but dot_sqp should not be abbreviated as dot since this would be ambiguous with other DOT method specifications.

In both the input specification and user input files, comments are preceded by #.

The dakota.input.summary file for DAKOTA version 5.1 is:

```plaintext
KEYWORD01 strategy
  [ graphics ]
  [ tabular_graphics_data
    [ tabular_graphics_file STRING ]
  ]
  [ output_precision INTEGER >= 0 ]
  [ iterator_servers INTEGER ]
  [ iterator_self_scheduling ]
  [ iterator_static_scheduling ]
  [ hybrid
    [ sequential ALIAS uncoupled
      method_list STRINGLIST ]
  ]
  [ embedded ALIAS coupled
    global_method_pointer STRING
    local_method_pointer STRING
    [ local_search_probability REAL ]
  ]
  [ collaborative
    method_list STRINGLIST ]

  [ multi_start
    method_pointer STRING
    [ random_starts INTEGER
      [ seed INTEGER ]
    ]
    [ starting_points REALLIST ]
  ]

  [ pareto_set
    method_pointer ALIAS opt_method_pointer STRING
    [ random_weight_sets INTEGER
      [ seed INTEGER ]
    ]
    [ weight_sets ALIAS multi_objective_weight_sets REALLIST ]
  ]

  [ single_method
    [ method_pointer STRING ]
  ]

KEYWORD12 method
```
[ id_method STRING ]
[ model_pointer STRING ]
[ output
debu DEBUG | verbose
| quiet
| silent
]
[ max_iterations INTEGER ]
[ max_function_evaluations INTEGER ]
[ speculative ]
[ convergence_tolerance REAL ]
[ constraint_tolerance REAL ]
[ scaling ]
[ final_solutions INTEGER >= 0 ]
[ surrogate_based_local
approx_method_name STRING
| approx_method_pointer STRING
[ soft_convergence_limit INTEGER ]
[ truth_surrogate_bypass ]
[ trust_region
| initial_size REAL ]
[ minimum_size REAL ]
[ contract_threshold REAL ]
[ expand_threshold REAL ]
[ contraction_factor REAL ]
[ expansion_factor REAL ]
]
[ approx_subproblem
original_primary
| single_objective
| augmented_lagrangian_objective
| lagrangian_objective
original_constraints
| linearized_constraints
| no_constraints
]
[ merit_function
penalty_merit
| adaptive_penalty_merit
| lagrangian_merit
| augmented_lagrangian_merit
]
[ acceptance_logic
tr_ratio
| filter
]
[ constraint_relax
homotopy
]
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ]}
[ optimization_type
  minimize
  | maximize
]
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ]
)
| dot_mmfd
| dot_bfgs
| dot_slp
| dot_sqp
|
| dot_frcg
| mmfd
| bfgs
| slp
| sqp
[ optimization_type
  minimize
  | maximize
]
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ]
)
|
| conmin_frcg
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ]
)
|
| conmin_mfd
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ]
2.1 dakota.input.summary File Reference

)

| dl_solver STRING
| [ linear_inequality_constraint_matrix REALIST ]
| [ linear_inequality_lower_bounds REALIST ]
| [ linear_inequality_upper_bounds REALIST ]
| [ linear_inequality_scale_types STRINGLIST ]
| [ linear_inequality_scales REALIST ]
| [ linear_equality_constraint_matrix REALIST ]
| [ linear_equality_targets REALIST ]
| [ linear_equality_scale_types STRINGLIST ]
| [ linear_equality_scales REALIST ]

| npsol_sqp
| [ verify_level INTEGER ]
| [ function_precision REAL ]
| [ linesearch_tolerance REAL ]
| [ linear_inequality_constraint_matrix REALIST ]
| [ linear_inequality_lower_bounds REALIST ]
| [ linear_inequality_upper_bounds REALIST ]
| [ linear_inequality_scale_types STRINGLIST ]
| [ linear_inequality_scales REALIST ]
| [ linear_equality_constraint_matrix REALIST ]
| [ linear_equality_targets REALIST ]
| [ linear_equality_scale_types STRINGLIST ]
| [ linear_equality_scales REALIST ]

| nlssol_sqp
| |
| stanford
| npsol
| nlssol
| [ verify_level INTEGER ]
| [ function_precision REAL ]
| [ linesearch_tolerance REAL ]
| [ linear_inequality_constraint_matrix REALIST ]
| [ linear_inequality_lower_bounds REALIST ]
| [ linear_inequality_upper_bounds REALIST ]
| [ linear_inequality_scale_types STRINGLIST ]
| [ linear_inequality_scales REALIST ]
| [ linear_equality_constraint_matrix REALIST ]
| [ linear_equality_targets REALIST ]
| [ linear_equality_scale_types STRINGLIST ]
| [ linear_equality_scales REALIST ]

| nlpql_sqp
| [ linear_inequality_constraint_matrix REALIST ]
| [ linear_inequality_lower_bounds REALIST ]
| [ linear_inequality_upper_bounds REALIST ]
| [ linear_inequality_scale_types STRINGLIST ]
| [ linear_inequality_scales REALIST ]
| [ linear_equality_constraint_matrix REALIST ]
| [ linear_equality_targets REALIST ]
| [ linear_equality_scale_types STRINGLIST ]
| [ linear_equality_scales REALIST ]

| optpp_cg
| [ max_step REAL ]
| [ gradient_tolerance REAL ]
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ]
)
| { optpp_q_newton
  | optpp_fd_newton
  | optpp_g_newton
  | optpp_newton
{ search_method
  value_based_line_search
  | gradient_based_line_search
  | trust_region
  | tr_pds
  ]
  [ merit_function STRING ]
  [ central_path STRING ]
  [ steplength_to_boundary REAL ]
  [ centering_parameter REAL ]
  [ max_step REAL ]
  [ gradient_tolerance REAL ]
  [ linear_inequality_constraint_matrix REALLIST ]
  [ linear_inequality_lower_bounds REALLIST ]
  [ linear_inequality_upper_bounds REALLIST ]
  [ linear_inequality_scale_types STRINGLIST ]
  [ linear_inequality_scales REALLIST ]
  [ linear_equality_constraint_matrix REALLIST ]
  [ linear_equality_targets REALLIST ]
  [ linear_equality_scale_types STRINGLIST ]
  [ linear_equality_scales REALLIST ]
) |
| { optpp_pds
  | search_scheme_size INTEGER ]
  [ linear_inequality_constraint_matrix REALLIST ]
  [ linear_inequality_lower_bounds REALLIST ]
  [ linear_inequality_upper_bounds REALLIST ]
  [ linear_inequality_scale_types STRINGLIST ]
  [ linear_inequality_scales REALLIST ]
  [ linear_equality_constraint_matrix REALLIST ]
  [ linear_equality_targets REALLIST ]
  [ linear_equality_scale_types STRINGLIST ]
  [ linear_equality_scales REALLIST ]
)
| { asynch_pattern_search ALIAS coliny_apps
  | initial_delta REAL ]
  | contraction_factor REAL ]
  | threshold_delta REAL ]
  | solution_target ALIAS solution_accuracy REAL ]
  [ synchronization
    blocking
    | nonblocking
  ]
  [ merit_function
    merit_max
  ]
2.1 dakota.input.summary File Reference

```plaintext
| merit_max_smooth |
| merit1 |
| merit1_smooth |
| merit2 |
| merit2_smooth |
| merit2_squared |

| constraint_penalty REAL |
| smoothing_factor REAL |
| linear_inequality_constraint_matrix REALLIST |
| linear_inequality_lower_bounds REALLIST |
| linear_inequality_upper_bounds REALLIST |
| linear_inequality_scale_types STRINGLIST |
| linear_inequality_scales REALLIST |
| linear_equality_constraint_matrix REALLIST |
| linear_equality_targets REALLIST |
| linear_equality_scale_types STRINGLIST |
| linear_equality_scales REALLIST |

| moga |
| fitness_type |
| layer_rank |
| domination_count |

| replacement_type |
| elitist |
| roulette_wheel |
| unique_roulette_wheel |

| below_limit REAL |
| shrinkage_fraction ALIAS shrinkage_percentage REAL |

| niching_type |
| radial REALLIST |
| distance REALLIST |

| convergence_type |
| metric_tracker |
| percent_change REAL |
| num_generations INTEGER >= 0 |

| postprocessor_type |
| orthogonal_distance REALLIST |

| population_size INTEGER |
| log_file STRING |
| print_each_pop |

| initialization_type |
| simple_random |
| unique_random |
| flat_file STRING |

| crossover_type |
| multi_point_binary INTEGER |
| multi_point_parameterized_binary INTEGER |
| multi_point_real INTEGER |

| shuffle_random |
| num_parents INTEGER > 0 |
| num_offspring INTEGER > 0 |
```
) } [ crossover_rate REAL ] } [ mutation_type
  bit_random
  | replace_uniform
  |
  ( offset_normal
    | offset_cauchy
    | offset_uniform
      { mutation_scale REAL } |
  ) [ mutation_rate REAL ] ]
] [ seed INTEGER ]
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ] )
|
{ soga
  [ fitness_type
    merit_function
      { constraint_penalty REAL } |
  ] [ replacement_type
    elitist
      { favor_feasible
        | roulette_wheel
          | unique_roulette_wheel
      |
    ]
  ] [ convergence_type
    { best_fitness_tracker
      [ percent_change REAL ]
        [ num_generations INTEGER >= 0 ] |
    ]
    |
      { average_fitness_tracker
        [ percent_change REAL ]
          [ num_generations INTEGER >= 0 ] |
    ]

  ] [ population_size INTEGER ]
[ log_file STRING ]
[ print_each_pop ]
[ initialization_type
  simple_random
    | unique_random
      | flat_file STRING |
  ]
[ crossover_type
  multi_point_binary INTEGER
    | multi_point_parameterized_binary INTEGER
    | multi_point_real INTEGER |
    |
    { shuffle_random
      | |
    }
[ num_parents INTEGER > 0 ]
( num_offspring INTEGER > 0 )
)
[ crossover_rate REAL ]
]
[ mutation_type
bit_random
| replace_uniform
|
( offset_normal
| offset_cauchy
| offset_uniform
( mutation_scale REAL ]
)
[ mutation_rate REAL ]
]
[ seed INTEGER ]
[ linear_inequality_constraint_matrix REALLIST ]
[ linear_inequality_lower_bounds REALLIST ]
[ linear_inequality_upper_bounds REALLIST ]
[ linear_inequality_scale_types STRINGLIST ]
[ linear_inequality_scales REALLIST ]
[ linear_equality_constraint_matrix REALLIST ]
[ linear_equality_targets REALLIST ]
[ linear_equality_scale_types STRINGLIST ]
[ linear_equality_scales REALLIST ]
)
|
( coliny_pattern_search
[ constant_penalty ]
[ no_expansion ]
[ expand_after_success INTEGER ]
[ pattern_basis
coordinate
| simplex
]
[ stochastic ]
[ total_pattern_size INTEGER ]
[ exploratory_moves
multi_step
| adaptive_pattern
| basic_pattern
]
[ synchronization
blocking
| nonblocking
]
[ contraction_factor REAL ]
[ constraint_penalty REAL ]
initial_delta REAL
threshold_delta REAL
[ solution_target ALIAS solution_accuracy REAL ]
[ seed INTEGER ]
[ show_misc_options ]
[ misc_options STRINGLIST ]
)
|
( coliny_solis_wets
[ contract_after_failure INTEGER ]
[ no_expansion ]
[ expand_after_success INTEGER ]
[ constant_penalty ]
[ contraction_factor REAL ]
[ constraint_penalty REAL ]
initial_delta REAL
threshold_delta REAL
[ solution_target ALIAS solution_accuracy REAL ]
[ seed INTEGER ]
[ show_m misc_options ]
[ misc_options STRINGLIST ]
)

|{ coliny_cobyla
initial_delta REAL
threshold_delta REAL
[ solution_target ALIAS solution_accuracy REAL ]
[ seed INTEGER ]
[ show_m misc_options ]
[ misc_options STRINGLIST ]
)

|{ coliny_direct
{ division
   major_dimension
   | all_dimensions
   }
[ global_balance_parameter REAL ]
[ local_balance_parameter REAL ]
[ max_boxsize_limit REAL ]
[ min_boxsize_limit REAL ]
[ constraint_penalty REAL ]
[ solution_target ALIAS solution_accuracy REAL ]
[ seed INTEGER ]
[ show_m misc_options ]
[ misc_options STRINGLIST ]
)

|{ coliny_ea
[ population_size INTEGER ]
[ initialization_type
   simple_random
   | unique_random
   | flat_file STRING
   ]
[ fitness_type
   linear_rank
   | merit_function
   ]
[ replacement_type
   random INTEGER
   | chc INTEGER
   | elitist INTEGER
   [ new_solutions_generated INTEGER ]
   ]
[ crossover_rate REAL ]
[ crossover_type
   two_point
   | blend
   | uniform
   ]
[ mutation_rate REAL ]
[ mutation_type
   replace_uniform
   |}
{ offset_normal
 | offset_cauchy
 | offset_uniform
 [ mutation_scale REAL ]
 [ mutation_range INTEGER ]

 | non_adaptive
 |
 | constraint_penalty REAL |
 | solution_target ALIAS solution_accuracy REAL |
 | seed INTEGER |
 | show_misc_options |
 [ misc_options STRINGLIST ]

 | nl2sol
 [ function_precision REAL ]
 [ absolute_conv_tol REAL ]
 [ x_conv_tol REAL ]
 [ singular_conv_tol REAL ]
 [ singular_radius REAL ]
 [ false_conv_tol REAL ]
 [ initial_trust_radius REAL ]
 [ covariance INTEGER ]
 [ regression_diagnostics ]
 |
 | nonlinear_cg
 [ misc_options STRINGLIST ]
 |
 | surrogate_based_global
 approx_method_name STRING
 | approx_method_pointer STRING
 [ replace_points ]
 |
 | efficient_global
 [ seed INTEGER ]
 |
 | polynomial_chaos ALIAS nond_polynomial_chaos
 [ p_refinement
 uniform |
 [ adaptive
 [ sobol
 | decay
 | generalized ]
 |
 | askey
 | wiener ]
 ( quadrature_order INTEGERLIST
 [ nested
 | non_nested ]
 )
 |
 ( sparse_grid_level INTEGER
 [ dimension_preference REALLIST ]
 [ nested
 | non_nested ]
{ cubature_integrand INTEGER
|
{ collocation_points INTEGER
| collocation_ratio REAL
| [ reuse_points ALIAS reuse_samples ]
| [ use_derivatives ]
| expansion_order INTEGERLIST
| expansion_terms INTEGER
}
|
{ expansion_samples INTEGER
| incremental_lhs
| expansion_order INTEGERLIST
| expansion_terms INTEGER
}
|
{ expansion_import_file STRING
| expansion_order INTEGERLIST
| expansion_terms INTEGER
}
|
[ variance_based_decomp
| [ univariate_effects ]
| [ drop_tolerance REAL ]
]
|
[ sample_refinement
| import
| adapt_import
| mm_adapt_import
]
|
[ sample_type
| lhs
| random
]
|
| [ all_variables ]
| [ fixed_seed ]
| [ reliability_levels REALLIST
| [ num_reliability_levels INTEGERLIST ]
]

| [ response_levels REALLIST
| [ num_response_levels INTEGERLIST ]
| [ compute
| probabilities
| reliabilities
| gen_reliabilities
| ]
]
|
| [ distribution
| cumulative
| complementary
]
|
| [ probability_levels REALLIST
| [ num_probability_levels INTEGERLIST ]
]

| [ gen_reliability_levels REALLIST
| [ num_gen_reliability_levels INTEGERLIST ]
]
|
[ rng
| mt19937
| rnum2
]
| [ samples INTEGER ]
}
[ seed INTEGER ]
)

[ stoch_collocation ALIAS nond_stoch_collocation
  [ p_refinement
    uniform
    |
    { adaptive
      [ sobol
        | generalized ]
    )
  ]
[ askey
  | wiener ]
quadrature_order INTEGERLIST
|
( sparse_grid_level INTEGER
  [ dimension_preference REALLIST ]
)
[ nested
  | non_nested ]
[ variance_based_decomp
  [ univariate_effects ]
  [ drop_tolerance REAL ]
]
sample_refinement
import
  | adapt_import
  | mm_adapt_import
]
sample_type
lhs
  | random
]
[ all_variables ]
[ fixed_seed ]
[ reliability_levels REALLIST
  [ num_reliability_levels INTEGERLIST ]
]
response_levels REALLIST
  [ num_response_levels INTEGERLIST ]
  [ compute
    probabilities
    | reliabilities
    | gen_reliabilities
  ]
]
distribution
  cumulative
  | complementary
]
probability_levels REALLIST
  [ num_probability_levels INTEGERLIST ]
]
gen_reliability_levels REALLIST
  [ num_gen_reliability_levels INTEGERLIST ]
]
rng
  mt19937
  | rnum2
]
samples INTEGER ]
[ seed INTEGER ]
)
|
{ sampling ALIAS nond_sampling
[ sample_type
    random
    lhs
]
( incremental_lhs
    incremental_random
    previous_samples INTEGER
)
]
[ variance_based_decomp
[ drop_tolerance REAL ]
]
[ all_variables ]
[ fixed_seed ]
[ reliability_levels REALLIST ]
{ num_reliability_levels INTEGERLIST }
[ response_levels REALLIST ]
{ num_response_levels INTEGERLIST }
[ compute
    probabilities
    reliabilities
    gen_reliabilities
]
[ distribution
    cumulative
    complementary
]
[ probability_levels REALLIST ]
{ num_probability_levels INTEGERLIST }
[ gen_reliability_levels REALLIST ]
{ num_gen_reliability_levels INTEGERLIST }
[ rng
    mt19937
    rnum2
]
[ samples INTEGER ]
[ seed INTEGER ]
)
|
{ importance_sampling ALIAS nond_importance_sampling
[ response_levels REALLIST ]
{ num_response_levels INTEGERLIST }
[ compute
    probabilities
    gen_reliabilities
]
[ distribution
    cumulative
    complementary
]
[ probability_levels REALLIST ]
{ num_probability_levels INTEGERLIST }
]


| dace
| grid
| random
| oas
| lhs
| oa_lhs
| box_behnken
| central_composite
| main_effects
| quality_metrics
| variance_based_decomp
| drop_tolerance
| fixed_seed
| symbols
| samples
| seed

| fsu_cvt
| latinize
| variance_based_decomp
| drop_tolerance
| fixed_seed
| num_trials
| samples
| seed

| psuade_moat
| partitions
| variance_based_decomp
| drop_tolerance
| fixed_seed
| num_trials
| samples
| seed

| ncsu_direct
| solution_target
| solution_accuracy
| min_boxsize_limit
| volume_boxsize_limit

| local_evidence
| solution_target
| solution_accuracy
| min_boxsize_limit
| volume_boxsize_limit
| sqp
| nip
| response_levels
| num_response_levels
| compute_probabilities
| gen_reliabilities
| probability_levels
| num_probability_levels

[ gen_reliability_levels REALLIST
  [ num_gen_reliability_levels INTEGERLIST ]
]
[ distribution
  cumulative
  | complementary
]
]
[ local_interval_est ALIAS nond_local_interval_est
  [ sqp
    | nip ]
]
[ local_reliability ALIAS nond_local_reliability
  [ mpp_search
    x_taylor_mean
    | u_taylor_mean
    | x_taylor_mpp
    | u_taylor_mpp
    | x_two_point
    | u_two_point
    | no_approx
    [ sqp
      | nip ]
  ]
[ integration
  first_order
  | second_order
  [ sample_refinement
    import
    | adapt_import
    | mm_adapt_import
    [ samples INTEGER ]
    [ seed INTEGER ]
  ]
][ response_levels REALLIST
  [ num_response_levels INTEGERLIST ]
][ compute
  probabilities
  | reliabilities
  | gen_reliabilities
  |
][ reliability_levels REALLIST
  [ num_reliability_levels INTEGERLIST ]
][ distribution
  cumulative
  | complementary
]
[ probability_levels REALLIST
  [ num_probability_levels INTEGERLIST ]
][ gen_reliability_levels REALLIST
  [ num_gen_reliability_levels INTEGERLIST ]
]
]}
x_gaussian_process
| u_gaussian_process
| [ all_variables ]
| [ seed INTEGER ]
| [ rng
| mt19937
| | rnum2
| ]
| [ response_levels REALIST
| [ num_response_levels INTEGERLIST ]
| [ compute
| probabilities
| ]
| ]
| [ distribution
| cumulative
| ]
| [ probability_levels REALIST
| [ num_probability_levels INTEGERLIST ]
| ]
| [ gen_reliability_levels REALIST
| [ num_gen_reliability_levels INTEGERLIST ]
| ]
| [ fsu_quasi_mc
| halton
| | hammersley
| | latinize ]
| [ quality_metrics ]
| [ variance_based_decomp
| [ drop_tolerance REAL ]
| ]
| [ samples INTEGER ]
| [ fixed_sequence ]
| [ sequence_start INTEGERLIST ]
| [ sequence_leap INTEGERLIST ]
| [ prime_base INTEGERLIST ]
| ]
| [ vector_parameter_study
| final_point REALIST
| | step_vector REALLIST
| num_steps INTEGER
| ]
| [ list_parameter_study
| list_of_points REALIST
| ]
| [ centered_parameter_study
| step_vector REALLIST
| steps_per_variable ALIAS deltas_per_variable INTEGERLIST
| ]
| [ multidim_parameter_study
| partitions INTEGERLIST
| ]
| [ richardson_extrap
| ]
estimate_order
| converge_order
| converge_qoi
[ refinement_rate REAL ]
)

KEYWORD model
[ id_model STRING ]
[ variables_pointer STRING ]
[ responses_pointer STRING ]
( single
[ interface_pointer STRING ]
)
|
| surrogate
[ id_surrogates INTEGERLIST ]
) global
{ gaussian_process
[ point_selection ]
[ trend
  constant
  | linear
  | reduced_quadratic
 ]
)
|
| mars
[ max_bases INTEGER ]
[ interpolation
  linear
  | cubic
 ]
)
|
| moving_least_squares
[ poly_order INTEGER ]
[ weight_function INTEGER ]
)
|
| neural_network
[ nodes INTEGER ]
[ range REAL ]
[ random_weight INTEGER ]
)
|
| radial_basis
[ bases INTEGER ]
[ max_pts INTEGER ]
[ min_partition INTEGER ]
[ max_subsets INTEGER ]
)
|
| polynomial
linear
| quadratic
| cubic
)
|
| kriging
[ trend
  constant
  | linear
)
| reduced_quadratic
| quadratic
|
| optimization_method STRING |
| max_trials INTEGER > 0 |
| correlation_lengths REALIST |
|
| total_points INTEGER
| minimum_points
| recommended_points |
| dace_method_pointer STRING |
| reuse_points ALIAS reuse_samples
  all
  | region
  | none
|
| points_file ALIAS samples_file STRING |
| use_derivatives |
| correction
  zeroth_order
  | first_order
  | second_order
  additive
  | multiplicative
  | combined
|
| diagnostics STRINGLIST |
|
| multipoint
  tana
  actual_model_pointer STRING |
|
| local
  taylor_series
  actual_model_pointer STRING |
|
| hierarchical
  low_fidelity_model_pointer STRING
  high_fidelity_model_pointer STRING
  ( correction
    zeroth_order
    | first_order
    | second_order
    additive
    | multiplicative
    | combined
  )
|
| nested
  ( optional_interface_pointer STRING
    | optional_interface_responses_pointer STRING ) |
|
| sub_method_pointer STRING
  ( primary_variable_mapping STRINGLIST |
    | secondary_variable_mapping STRINGLIST |
    | primary_response_mapping REALLIST |
    | secondary_response_mapping REALLIST )
KEYWORD12 variables

  [ id_variables STRING ]
  [ continuous_design INTEGER > 0
    [ initial_point ALIAS cdv_initial_point REALIST ]
    [ lower_bounds ALIAS cdv_lower_bounds REALIST ]
    [ upper_bounds ALIAS cdv_upper_bounds REALIST ]
    [ scale_types ALIAS cdv_scale_types STRINGLIST ]
    [ scales ALIAS cdv_scales REALIST ]
    [ descriptors ALIAS cdv_descriptors STRINGLIST ]
  ]

  [ discrete_design_range INTEGER > 0
    [ initial_point ALIAS ddv_initial_point INTEGERLIST ]
    [ lower_bounds ALIAS ddv_lower_bounds INTEGERLIST ]
    [ upper_bounds ALIAS ddv_upper_bounds INTEGERLIST ]
    [ descriptors ALIAS ddv_descriptors STRINGLIST ]
  ]

  [ discrete_design_set_integer INTEGER > 0
    [ initial_point INTEGERLIST ]
    [ num_set_values INTEGERLIST ]
    [ set_values INTEGERLIST ]
    [ descriptors STRINGLIST ]
  ]

  [ discrete_design_set_real INTEGER > 0
    [ initial_point REALIST ]
    [ num_set_values INTEGERLIST ]
    [ set_values REALIST ]
    [ descriptors STRINGLIST ]
  ]

  [ normal_uncertain INTEGER > 0
    means ALIAS nuv_means REALIST
    std_deviations ALIAS nuv_std_deviations REALIST
    [ lower_bounds ALIAS nuv_lower_bounds REALIST ]
    [ upper_bounds ALIAS nuv_upper_bounds REALIST ]
    [ descriptors ALIAS nuv_descriptors STRINGLIST ]
  ]

  [ lognormal_uncertain INTEGER > 0
    [ lambdas ALIAS lnuv_lambdas REALIST
      zetas ALIAS lnuv_zetas REALIST
    ]
    [ means ALIAS lnuv_means REALIST
      std_deviations ALIAS lnuv_std_deviations REALIST
      error_factors ALIAS lnuv_error_factors REALIST
    ]
    [ lower_bounds ALIAS lnuv_lower_bounds REALIST ]
    [ upper_bounds ALIAS lnuv_upper_bounds REALIST ]
    [ descriptors ALIAS lnuv_descriptors STRINGLIST ]
  ]

  [ uniform_uncertain INTEGER > 0
    lower_bounds ALIAS uuv_lower_bounds REALIST
    upper_bounds ALIAS uuv_upper_bounds REALIST
    [ descriptors ALIAS uuv_descriptors STRINGLIST ]
  ]

  [ loguniform_uncertain INTEGER > 0
    lower_bounds ALIAS luuv_lower_bounds REALIST
    upper_bounds ALIAS luuv_upper_bounds REALIST
    [ descriptors ALIAS luuv_descriptors STRINGLIST ]
  ]

  [ triangular_uncertain INTEGER > 0
modes ALIAS tuv_modes REALLIST
lower_bounds ALIAS tuv_lower_bounds REALLIST
upper_bounds ALIAS tuv_upper_bounds REALLIST
[ descriptors ALIAS tuv_descriptors STRINGLIST ]
]
[ exponential_uncertain INTEGER > 0
betas ALIAS euv_betas REALLIST
[ descriptors ALIAS euv_descriptors STRINGLIST ]
]
[ beta_uncertain INTEGER > 0
alphas ALIAS buv_alphas REALIST
betas ALIAS buv_betas REALIST
lower_bounds ALIAS buv_lower_bounds REALIST
upper_bounds ALIAS buv_upper_bounds REALIST
[ descriptors ALIAS buv_descriptors STRINGLIST ]
]
[ gamma_uncertain INTEGER > 0
alphas ALIAS gauv_alphas REALIST
betas ALIAS gauv_betas REALIST
[ descriptors ALIAS gauv_descriptors STRINGLIST ]
]
[ frechet_uncertain INTEGER > 0
alphas ALIAS fuv_alphas REALIST
betas ALIAS fuv_betas REALIST
[ descriptors ALIAS fuv_descriptors STRINGLIST ]
]
[ weibull_uncertain INTEGER > 0
alphas ALIAS wuv_alphas REALIST
betas ALIAS wuv_betas REALIST
[ descriptors ALIAS wuv_descriptors STRINGLIST ]
]
[ histogram_bin_uncertain INTEGER > 0
abscissas ALIAS huv_bin_abscissas REALIST
ordinates ALIAS huv_bin_ordinates REALIST
counts ALIAS huv_bin_counts REALIST
[ descriptors ALIAS huv_bin_descriptors STRINGLIST ]
]
[ poisson_uncertain INTEGER > 0
lambdas REALIST
[ descriptors STRINGLIST ]
]
[ binomial_uncertain INTEGER > 0
prob_per_trial REALIST
num_trials INTEGERLIST
[ descriptors STRINGLIST ]
]
[ negative_binomial_uncertain INTEGER > 0
prob_per_trial REALIST
num_trials INTEGERLIST
[ descriptors STRINGLIST ]
]
[ geometric_uncertain INTEGER > 0
prob_per_trial REALIST
[ descriptors STRINGLIST ]
]
[ hypergeometric_uncertain INTEGER > 0

total_population INTEGERLIST
selected_population INTEGERLIST
num_drawn INTEGERLIST

[ descriptors STRINGLIST ]
]

[ histogram_point_uncertain INTEGER > 0
 [ num_pairs ALIAS huv_num_point_pairs INTEGERLIST ]
 abscissas ALIAS huv_point_abscissas REALLIST
counts ALIAS huv_point_counts REALLIST
 [ descriptors ALIAS huv_point_descriptors STRINGLIST ]
]

[ uncertain_correlation_matrix REALLIST ]

[ interval_uncertain INTEGER > 0
 [ num_intervals ALIAS iuv_num_intervals INTEGERLIST ]
 interval_probs ALIAS iuv_interval_probs REALLIST
 interval_bounds ALIAS iuv_interval_bounds REALLIST
 [ descriptors ALIAS iuv_descriptors STRINGLIST ]
]

[ continuous_state INTEGER > 0
 [ initial_state ALIAS csv_initial_state REALLIST ]
 lower_bounds ALIAS csv_lower_bounds REALLIST
 upper_bounds ALIAS csv_upper_bounds REALLIST
 [ descriptors ALIAS csv_descriptors STRINGLIST ]
]

[ discrete_state_range INTEGER > 0
 [ initial_state ALIAS dsv_initial_state INTEGERLIST ]
 lower_bounds ALIAS dsv_lower_bounds INTEGERLIST
 upper_bounds ALIAS dsv_upper_bounds INTEGERLIST
 [ descriptors ALIAS dsv_descriptors STRINGLIST ]
]

[ discrete_state_set_integer INTEGER > 0
 [ initial_state INTEGERLIST ]
 [ num_set_values INTEGERLIST ]
 set_values INTEGERLIST
 [ descriptors STRINGLIST ]
]

[ discrete_state_set_real INTEGER > 0
 [ initial_state REALLIST ]
 [ num_set_values INTEGERLIST ]
 set_values REALLIST
 [ descriptors STRINGLIST ]
]

KEYWORD12 interface

[ id_interface STRING ]
[ algebraic_mappings STRING ]
[ analysis_drivers STRINGLIST ]
[ analysis_components STRINGLIST ]
[ input_filter STRING ]
[ output_filter STRING ]

( system
 | fork
[ parameters_file STRING ]
[ results_file STRING ]
[ allow_existing_results ]
[ verbatim ]
[ aprepro ]
[ file_tag ]
[ file_save ]
[ work_directory ]
[ named STRING ]
[ directory_tag ALIAS dir_tag ]

[ directory_save ALIAS dir_save ]
[ template_directory STRING
  | template_files STRINGLIST
  | copy ]
  | replace ]
]
)
|
| direct
| [ processors_per_analysis INTEGER ]
|
| grid
| failure_capture
  abort
  | retry INTEGER
  | recover REALLIST
  | continuation
  |
| deactivate
| [ active_set_vector ]
| [ evaluation_cache ]
| [ restart_file ]
|
|
[ asynchronous
| [ evaluation_concurrency INTEGER ]
| [ local_evaluation_self_scheduling
  | local_evaluation_static_scheduling ]
| [ analysis_concurrency INTEGER ]
]
| [ evaluation_servers INTEGER ]
| [ evaluation_self_scheduling
  | evaluation_static_scheduling ]
| [ analysis_servers INTEGER ]
| [ analysis_self_scheduling
  | analysis_static_scheduling ]

KEYWORD12 responses
[ id_responses STRING ]
[ descriptors ALIAS response_descriptors STRINGLIST ]
| [ num_objective_functions INTEGER >= 0
  | objective_function_scale_types STRINGLIST ]
| [ objective_function_scales REALLIST ]
| [ multi_objective_weights REALLIST ]
| [ num_nonlinear_inequality_constraints INTEGER >= 0
  | nonlinear_inequality_lower_bounds REALLIST ]
| [ nonlinear_inequality_upper_bounds REALLIST ]
| [ nonlinear_inequality_scale_types STRINGLIST ]
| [ nonlinear_inequality_scales REALLIST ]
|
| [ num_nonlinear_equality_constraints INTEGER >= 0
  | nonlinear_equality_targets REALLIST ]
| [ nonlinear_equality_scale_types STRINGLIST ]
| [ nonlinear_equality_scales REALLIST ]
|
| [ num_least_squares_terms INTEGER >= 0
  | least_squares_data_file STRING ]
| [ least_squares_term_scale_types STRINGLIST ]
| [ least_squares_term_scales REALLIST ]
[ least_squares_weights REALLIST ]
[num_nonlinear_inequality_constraints INTEGER >= 0
[ nonlinear_inequality_lower_bounds REALLIST ]
[ nonlinear_inequality_upper_bounds REALLIST ]
[ nonlinear_inequality_scale_types STRINGLIST ]
[ nonlinear_inequality_scales REALLIST ]
]
[num_nonlinear_equality_constraints INTEGER >= 0
[ nonlinear_equality_targets REALLIST ]
[ nonlinear_equality_scale_types STRINGLIST ]
[ nonlinear_equality_scales REALLIST ]
]
)
)
num_response_functions INTEGER >= 0
no_gradients
analytic_gradients
)
mixed_gradients
id_numerical_gradients INTEGERLIST
id_analytic_gradients INTEGERLIST
[ method_source ]
{ dakota
  [ ignore_bounds ]
  }
vendor
interval_type
forward
central
fd_step_size ALIAS fd_gradient_step_size REALLIST := 0.001
)
)
umerical_gradients
[ method_source ]
{ dakota
  [ ignore_bounds ]
  }
vendor
interval_type
forward
central
fd_step_size ALIAS fd_gradient_step_size REALLIST := 0.001
)
)
umerical_hessians
[ fd_step_size ALIAS fd_hessian_step_size REALLIST ]
[ forward
  | central ]
)
)
quasi_hessians
( bfgs
  [ damped ]
  )
| sr1
| analytic_hessians
|mixed_hessians
[ id_numerical_hessians INTEGERLIST
  [ fd_step_size ALIAS fd_hessian_step_size REALLIST ]
  ]
[ forward
| central ]
[ id_quasi_hessians INTEGERLIST
  { bfgs
    [ damped ]
  }
| sr1
]
[ id_analytic_hessians INTEGERLIST ]
}
Chapter 3

Introduction

3.1 Overview

In DAKOTA, a strategy creates and manages iterators and models. A model, generally speaking, contains a set of variables, an interface, and a set of responses. An iterator repeatedly operates on the model to map the variables into responses using the interface. Each of these six pieces (strategy, method, model, variables, interface, and responses) are separate specifications in the user’s input file, and as a whole, determine the study to be performed during an execution of the DAKOTA software. A DAKOTA execution is limited to a single strategy, which may invoke multiple methods. Furthermore, each method may have its own model, consisting of (generally speaking) its own variables, interface, and set of responses. Thus, there may be multiple specifications of the method, model, variables, interface, and responses sections.

DAKOTA input is most commonly specified through a text file, whose syntax is governed by the New Input Deck Reader (NIDR) parsing system [Gay, 2008], in conjunction with the dakota.input.nspec file which describes allowable inputs. A more concise version of the input specification file, dakota.input.summary, offers a quick reference to keywords and data values from which a particular input file (e.g., dakota.in) may be derived. This automatically derived shortened form omits implementation details not needed in a quick reference.

This Reference Manual focuses on the details of allowable specifications in a file input to the DAKOTA program. Related details on the name and location of the dakota program, command line inputs, and execution syntax are provided in the Users Manual [Adams et al., 2010].

3.2 NIDR Input Specification File

DAKOTA input is governed by the NIDR input specification file. This file (dakota.input.nspec) is used by a code generator to create parsing system components that are compiled into the DAKOTA executable (refer to Instructions for Modifying DAKOTA’s Input Specification for additional information). Therefore, dakota.input.nspec and its derived summary, dakota.input.summary, are the definitive source for input syntax, capability options, and optional and required capability sub-parameters for any given DAKOTA version. Beginning users may find dakota.input.summary overwhelming or confusing and will likely derive more benefit from adapting example input files to a particular problem. However, advanced users can master the many input specification possibilities by understanding the structure of the input specification file.
Refer to the dakota.input.summary file for current input specifications. From this file listing, it can be seen that the main structure of the strategy specification consists of several required group specifications separated by logical OR’s (indicated by \(|\) ); either hybrid | multi-start | pareto set OR single method. The method keyword is the most lengthy specification; however, its structure is again relatively simple. The structure is a set of optional method-independent settings followed by a long list of possible methods appearing as required group specifications (containing a variety of method-dependent settings) separated by OR’s. The model keyword reflects a structure of three required group specifications separated by OR’s. Within the surrogate model type, the type of approximation must be specified with either a global | multipoint | local | hierarchical required group specification. The structure of the variables keyword is that of optional group specifications for continuous and discrete design variables, a number of different uncertain variable distribution types, and continuous and discrete state variables. Each of these specifications can either appear or not appear as a group. Next, the interface keyword allows the specification of either algebraic mappings, simulation-based analysis driver mappings, or both. Within the analysis drivers specification, a system | fork | direct | grid group specification must be selected. Finally, within the responses keyword, the primary structure is the required specification of the function set (either optimization functions | least squares functions | generic response functions), followed by the required specification of the gradients (either none | numerical | analytic | mixed) and the required specification of the Hessians (either none | numerical | quasi | analytic | mixed). Refer to Strategy Commands, Method Commands, Model Commands, Variables Commands, Interface Commands, and Responses Commands for detailed information on the keywords and their various optional and required specifications. And for additional details on NIDR specification logic and rules, refer to [Gay, 2008].

Some keywords, such as those providing bounds on variables, have an associated list of values. When the same value should be repeated several times in a row, you can use the notation \(N\ast value\) instead of repeating the value \(N\) times. For example, in Sample 2: Least Squares below,

```
lower_bounds -2.0 -2.0
upper_bounds  2.0  2.0
```

could also be written

```
lower_bounds  2\ast-2.0
upper_bounds  2 \ast  2.0
```

(with optional spaces around the \(\ast\)). Another possible abbreviation is for sequences: L:S:U (with optional spaces around the \(:\)) is expanded to L L+S L+2+S ... U, and L:U (with no second colon) is treated as L:1:U. For example, in one of the test examples distributed with DAKOTA (test case 2 of test/dakota_uq_textbook_sop_lhs.in),

```
histogram_point = 2
abscissas = 50. 60. 70. 80. 90. 30. 40. 50. 60. 70.
counts = 10 20 30 20 10 10 20 30 20 10
```

could also be written

```
histogram_point = 2
abscissas = 50 : 10 : 90
            30 : 10 : 70
counts = 10:10:30 20 10
         10:10:30 20 10
```
3.3 Common Specification Mistakes

Spelling mistakes and omission of required parameters are the most common errors. Some causes of errors are more obscure:

- Documentation of new capability sometimes lags its availability in source and executables, especially nightly releases. When parsing errors occur that the documentation cannot explain, reference to the particular input specification used in building the executable, which is installed alongside the executable, will often resolve the errors.

- If you want to compare results with those obtained using an earlier version of DAKOTA (prior to 4.1), your input file for the earlier version must use backslashes to indicate continuation lines for DAKOTA keywords. For example, rather than

```plaintext
# Comment about the following "responses" keyword...
responses,
  num_objective_functions = 1
# Comment within keyword "responses"
  analytic_gradients
# Another comment within keyword "responses"
  no_hessians
```

you would need to write

```plaintext
# Comment about the following "responses" keyword...
responses,
  num_objective_functions = 1
    # Comment within keyword "responses"
    analytic_gradients
    # Another comment within keyword "responses"
    no_hessians
```

with no white space (blanks or tabs) after the \ character.

In most cases, the NIDR system provides error messages that help the user isolate errors in DAKOTA input files.

3.4 Sample dakota.in Files

A DAKOTA input file is a collection of fields from the dakota.input.summary file that describe the problem to be solved by DAKOTA. Several examples follow.

3.4.1 Sample 1: Optimization

The following sample input file shows single-method optimization of the Textbook Example using DOT’s modified method of feasible directions. A similar file is available as Dakota/examples/tutorial/dakota_textbook.in.
40 Introduction

strategy,
    single_method

method,
    # DOT performs better, but may not be available
dot_mmfd,
    # conmin_mfd,
    max_iterations = 50,
    convergence_tolerance = 1e-4

variables,
    continuous_design = 2
    initial_point 0.9 1.1
    upper_bounds 5.8 2.9
    lower_bounds 0.5 -2.9
    descriptors 'x1' 'x2'

interface,
    direct
    analysis_driver = 'text_book'

responses,
    num_objective_functions = 1
    num_nonlinear_inequality_constraints = 2
    numerical_gradients
        method_source dakota
        interval_type central
        fd_gradient_step_size = 1.e-4
    no_hessians

3.4.2 Sample 2: Least Squares

The following sample input file shows a nonlinear least squares solution of the Rosenbrock Example using the NL2SOL method. A similar file is available as Dakota/examples/tutorial/dakota_rosenbrock_-ls.in.

strategy,
    single_method

method,
    nl2sol
    max_iterations = 50
    convergence_tolerance = 1e-4

model,
    single

variables,
    continuous_design = 2
    initial_point -1.2 1.0
    lower_bounds -2.0 -2.0
    upper_bounds 2.0 2.0
    descriptor 'x1' 'x2'

interface,
    system
    analysis_driver = 'rosenbrock'

responses,
3.4 Sample dakota.in Files

3.4.3 Sample 3: Nondeterministic Analysis

The following sample input file shows Latin Hypercube Monte Carlo sampling using the Textbook Example. A similar file is available as Dakota/test/dakota_uq_textbook_lhs.in.

```plaintext
strategy,
  single_method
method,
  sampling,
    samples = 100 seed = 1
    complementary distribution
    response_levels = 3.6e+11 4.0e+11 4.4e+11
                     6.0e+04 6.5e+04 7.0e+04
                     3.5e+05 4.0e+05 4.5e+05
    sample_type lhs
variables,
  normal_uncertain = 2
    means = 248.89, 593.33
    std_deviations = 12.4, 29.7
    descriptors = ‘TF1n’ ‘TF2n’
  uniform_uncertain = 2
    lower_bounds = 199.3, 474.63
    upper_bounds = 298.5, 712.
    descriptors = ‘TF1u’ ‘TF2u’
  weibull_uncertain = 2
    alphas = 12., 30.
    betas = 250., 590.
    descriptors = ‘TF1w’ ‘TF2w’
histogram_bin_uncertain = 2
  num_pairs = 3 4
  abscissas = 5 8 10 .1 .2 .3 .4
  counts = 17 21 0 12 24 12 0
  descriptors = ‘TF1h’ ‘TF2h’
histogram_point_uncertain = 1
  num_pairs = 2
  abscissas = 3 4
  counts = 1 1
  descriptors = ‘TF3h’
interface,
  system asynch evaluation_concurrency = 5
  analysis_driver = ‘text_book’
responses,
  num_response_functions = 3
  no_gradients
  no_hessians
```
3.4.4 Sample 4: Parameter Study

The following sample input file shows a 1-D vector parameter study using the Textbook Example. It makes use of the default strategy and model specifications (single_method and single, respectively), so they can be omitted. A similar file is available in the test directory as Dakota/examples/tutorial/dakota_rosenbrock_vector.in.

```dakota
strategy,
  single_method

method,
  vector_parameter_study
    final_point = 1.1 1.3
    num_steps = 10

model,
  single

variables,
  continuous_design = 2
    initial_point = -0.3 0.2
    descriptors = 'x1' "x2"

interface,
  direct
    analysis_driver = 'rosenbrock'

responses,
  num_objective_functions = 1
  no_gradients
  no_hessians
```

3.4.5 Sample 5: Hybrid Strategy

The following sample input file shows a hybrid strategy using three methods. It employs a genetic algorithm, pattern search, and full Newton gradient-based optimization in succession to solve the Textbook Example. A similar file is available as Dakota/examples/methods/dakota_hybrid.in.

```dakota
strategy,
  graphics
    hybrid_sequential
      method_list = 'GA' 'PS' 'NLP'

method,
  id_method = 'GA'
  model_pointer = 'M1'
  coliny_ea
    seed = 1234
    population_size = 10
    verbose output

method,
  id_method = 'PS'
  model_pointer = 'M1'
  coliny_pattern_search stochastic
    seed = 1234
    initial_delta = 0.1
```
threshold_delta = 1.e-4
solution_accuracy = 1.e-10
exploratory_moves basic_pattern
verbose output

method,
id_method = 'PS2'
model_pointer = 'M1'
max_function_evaluations = 10
coliny_pattern_search stochastic
seed = 1234
initial_delta = 0.1
threshold_delta = 1.e-4
solution_accuracy = 1.e-10
exploratory_moves basic_pattern
verbose output

model,
id_model = 'M1'
single
variables_pointer = 'V1'
interface_pointer = 'I1'
responses_pointer = 'R1'

model,
id_model = 'M2'
single
variables_pointer = 'V1'
interface_pointer = 'I1'
responses_pointer = 'R2'

variables,
id_variables = 'V1'
continuous_design = 2
initial_point 0.6 0.7
upper_bounds 5.8 2.9
lower_bounds 0.5 -2.9
descriptors 'x1' 'x2'

interface,
id_interface = 'I1'
direct
analysis_driver = 'text_book'

responses,
id_responses = 'R1'
um_objective_functions = 1
no_gradients
no_hessians

responses,
id_responses = 'R2'
um_objective_functions = 1
analytic_gradients
Additional example input files, as well as the corresponding output and graphics, are provided in the Tutorial chapter of the Users Manual [Adams et al., 2010].

### 3.5 Tabular descriptions

In the following discussion of keyword specifications, tabular formats (Tables 4.1 through 9.10) are used to present a short description of the specification, the keyword used in the specification, the type of data associated with the keyword, the status of the specification (required, optional, required group, or optional group), and the default for an optional specification.

It can be difficult to capture in a simple tabular format the complex relationships that can occur when specifications are nested within multiple groupings. For example, in the model keyword, the actual_model_pointer specification is a required specification within the multipoint and local required group specifications, which are separated from each other and from other required group specifications (global and hierarchical) by logical OR’s. The selection between the global, multipoint, local, or hierarchical required groups is contained within another required group specification (surrogate), which is separated from the single and nested required group specifications by logical OR’s. Rather than unnecessarily proliferate the number of tables in attempting to capture all of these inter-relationships, a balance is sought, since some inter-relationships are more easily discussed in the associated text. The general structure of the following sections is to present the outermost specification groups first (e.g., single, surrogate, or nested in Table 6.1), followed by lower levels of group specifications (e.g., global, multipoint, local, or hierarchical surrogates in Table 6.3), followed by the components of each group (e.g., Tables 6.4 through 6.8) in succession.
Chapter 4

Strategy Commands

4.1 Strategy Description

The strategy section in a DAKOTA input file specifies the top level technique which will govern the management of iterators and models in the solution of the problem of interest. Four strategies currently exist: hybrid, multi_start, pareto_set, and single_method. These algorithms are implemented within the Strategy class hierarchy in the CollaborativeHybridStrategy, EmbeddedHybridStrategy, SequentialHybridStrategy, ConcurrentStrategy, and SingleMethodStrategy classes. For each of the strategies, a brief algorithm description is given below. Additional information on the algorithm logic is available in the Users Manual [Adams et al., 2010].

In a hybrid minimization strategy (hybrid), a set of methods synergistically seek an optimal design. The relationships among the methods are categorized as collaborative, embedded, or sequential. The goal in each case is to exploit the strengths of different optimization and nonlinear least squares algorithms through different stages of the minimization process. Global/local hybrids (e.g., genetic algorithms combined with nonlinear programming) are a common example in which the desire for identification of a global optimum is balanced with the need for efficient navigation to a local optimum.

In the multi-start iteration strategy (multi_start), a series of iterator runs are performed for different values of parameters in the model. A common use is for multi-start optimization (i.e., different local optimization runs from different starting points for the design variables), but the concept and the code are more general. An important feature is that these iterator runs may be performed concurrently.

In the pareto set optimization strategy (pareto_set), a series of optimization or least squares calibration runs are performed for different weightings applied to multiple objective functions. This set of optimal solutions defines a "Pareto set," which is useful for investigating design trade-offs between competing objectives. Again, these optimizations can be performed concurrently, similar to the multi-start strategy discussed above. The code is similar enough to the multi_start technique that both strategies are implemented in the same ConcurrentStrategy class.

Lastly, the single_method strategy is the default strategy in that it does not provide control over multiple iterators or multiple models. Rather, it provides the means for simple execution of a single iterator on a single model. It is most commonly used when specifying other strategy-independent controls, e.g., tabular output, where the existence of the strategy block mandates selection of a strategy.

Each of the strategy specifications identifies one or more method pointers (e.g., method_list, method_
pointer) to identify the iterators that will be used in the strategy. These method pointers are strings that correspond to the id_method identifier strings from the method specifications (see Method Independent Controls). These string identifiers (e.g., ‘NLP1’) should not be confused with method selections (e.g., dot_mmfd). Each of the method specifications identified in this manner has the responsibility for identifying corresponding model specifications (using model_pointer from Method Independent Controls), which in turn identify the variables, interface, and responses specifications (using variables_pointer, interface_pointer, and responses_pointer from Model Commands) that are used to build the model used by the iterator. If one of these specifications does not provide an optional pointer, then that component will be constructed using the last specification parsed. In addition to method pointers, a variety of graphics options (e.g., tabular_graphics_data), iterator concurrency controls (e.g., iterator_servers), and strategy data (e.g., starting_points) can be specified.

Specification of a strategy block in an input file is optional, with single_method being the default strategy. If no strategy is specified or if single_method is specified without its optional method_pointer specification, then the default behavior is to employ the last method, variables, interface, and responses specifications parsed. This default behavior is most appropriate if only one specification is present for method, variables, interface, and responses, since there is no ambiguity in this case.

Example specifications for each of the strategies follow. A hybrid example is:

```plaintext
strategy,
hybrid sequential
  method_list = 'GA', 'PS', 'NLP'
```

A multi_start example specification is:

```plaintext
strategy,
multi_start
  method_pointer = 'NLP1'
  random_starts = 10
```

A pareto_set example specification is:

```plaintext
strategy,
pareto_set
  method_pointer = 'NLP1'
  random_weight_sets = 10
```

And finally, a single_method example specification is:

```plaintext
strategy,
single_method
  method_pointer = 'NLP1'
```

### 4.2 Strategy Specification

The strategy specification has the following structure:

```plaintext
strategy,
<strategy independent controls>
<strategy selection>
  <strategy dependent controls>
```
where `<strategy selection>` is one of the following: hybrid, multi_start, pareto_set, or single_method.

The `<strategy independent controls>` are those controls which are valid for a variety of strategies. Unlike the Method Independent Controls, which can be abstractions with slightly different implementations from one method to the next, the implementations of each of the strategy independent controls are consistent for all strategies that use them. The `<strategy dependent controls>` are those controls which are only meaningful for a specific strategy. Referring to dakota.input.summary, the strategy independent controls are those controls defined externally from and prior to the strategy selection blocks. They are all optional. The strategy selection blocks are all required group specifications separated by logical OR's (hybrid OR multi_start OR pareto_set OR single_method). Thus, one and only one strategy selection must be provided. The strategy dependent controls are those controls defined within the strategy selection blocks. Defaults for strategy independent and strategy dependent controls are defined in DataStrategy. The following sections provide additional detail on the strategy independent controls followed by the strategy selections and their corresponding strategy dependent controls.

### 4.3 Strategy Independent Controls

The strategy independent controls are summarized in Table 4.1, including graphics, output data, and iterator scheduling controls. The graphics flag activates a 2D graphics window containing history plots for the variables and response functions in the study. This window is updated in an event loop with approximately a 2 second cycle time. The tabular_graphics_data flag activates file tabulation of the same variables and response function history data that gets passed to graphics windows with use of the graphics flag. The tabular_graphics_file specification optionally specifies a name to use for this file (dakota_tabular.dat is the default). Within the file, the variables and response functions appear as columns and each function evaluation provides a new table row. This capability is most useful for post-processing of DAKOTA results with third-party graphics tools such as MATLAB, Tecplot, etc. There is no dependence between the graphics flag and the tabular_graphics_data flag; they may be used independently or concurrently. Numeric output precision (as passed to the C++ control std::setprecision) can be controlled with output_precision with an upper limit of 16 enforced at run-time. When this control is not specified, most DAKOTA output will default to a precision of 10, though filesystem interfaces and pre-run output use higher precision for accuracy and better results reproducibility.

The iterator_servers, iterator_self_scheduling, and iterator_static_scheduling specifications provide manual overrides for the number of concurrent iterator partitions and the scheduling policy for concurrent iterator jobs. These settings are normally determined automatically in the parallel configuration routines (see ParallelLibrary) but can be overridden with user inputs if desired. The graphics, tabular_graphics_data, and tabular_graphics_file specifications are valid for all strategies. However, the iterator_servers, iterator_self_scheduling, and iterator_static_scheduling overrides are only useful inputs for those strategies supporting concurrency in iterators, i.e., multi_start and pareto_set.

### 4.4 Hybrid Minimization Commands

The hybrid minimization strategy has sequential, embedded, and collaborative approaches (see the Users Manual [Adams et al., 2010] for more information on the algorithms employed). In the sequential approach, the best solutions are transferred from one method to the next through a specified sequence. In the embedded
Table 4.1: Specification detail for strategy independent controls

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphics flag</td>
<td>graphics</td>
<td>none</td>
<td>Optional</td>
<td>no graphics</td>
</tr>
<tr>
<td>Tabulation of graphics data</td>
<td>tabular_graphics_</td>
<td>none</td>
<td>Optional group</td>
<td>no data tabulation</td>
</tr>
<tr>
<td>File name for tabular graphics data</td>
<td>tabular_graphics_</td>
<td>string</td>
<td>Optional</td>
<td>dakota_tabular.dat</td>
</tr>
<tr>
<td>Numeric output precision</td>
<td>output_precision</td>
<td>integer</td>
<td>Optional</td>
<td>10 (for most output)</td>
</tr>
<tr>
<td>Number of iterator servers</td>
<td>iterator_servers</td>
<td>integer</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
<tr>
<td>Self-scheduling of iterator jobs</td>
<td>iterator_self_</td>
<td>none</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
<tr>
<td>Static scheduling of iterator jobs</td>
<td>iterator_static_</td>
<td>none</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
</tbody>
</table>

Table 4.2: Specification detail for sequential hybrid strategies

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid strategy</td>
<td>hybrid</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Sequential hybrid</td>
<td>sequential</td>
<td>none</td>
<td>Required group (1 of 3 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>List of methods</td>
<td>method_list</td>
<td>list of strings</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

In the sequential approach, a list of method strings supplied with the method_list specification specifies the identity and sequence of iterators to be used. Any number of iterators may be specified. In the sequential approach, method switching is managed through the separate convergence controls of each method. The number of solutions transferred between methods is specified by the particular method through its final_solutions method control. For example, if one sets up a two-level strategy with a first method that generates multiple solutions such as a genetic algorithm, followed by a second method that is initialized only at a single point such as a gradient-based algorithm, it is possible to take the multiple solutions generated by the first method and create several instances of the second method, each one with a different initial starting point. The logic governing the transfer of multiple solutions between methods is as follows: if one solution is returned from method A, then one solution is transferred to method B. If multiple solutions are returned from method A, and method B can accept multiple solutions as input (for example, as a genetic algorithm population), then one instance of method B is initialized with multiple solutions. If multiple solutions are returned from method A but method B only can accept one initial starting point, then method B is run several times, each one with a separate starting point from the results of method A. Table 4.2 summarizes the sequential hybrid strategy inputs.

In the embedded approach, global and local method strings supplied with the global_method_pointer...
and local_method_pointer specifications identify the two methods to be used. The local_search_probability setting is an optional specification for supplying the probability (between 0.0 and 1.0) of employing local search to improve estimates within the global search. Table 4.3 summarizes the embedded hybrid strategy inputs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid strategy</td>
<td>hybrid</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Embedded hybrid</td>
<td>embedded</td>
<td>none</td>
<td>Required group (1 of 3 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Pointer to the global method</td>
<td>global_method_pointer</td>
<td>string</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>specification</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointer to the local method</td>
<td>local_method_pointer</td>
<td>string</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>specification</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of executing local</td>
<td>local_search_probability</td>
<td>real</td>
<td>Optional</td>
<td>0.1</td>
</tr>
<tr>
<td>searches</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Specification detail for embedded hybrid strategies

In the collaborative approach, a list of method strings supplied with the method_list specification specifies the pool of iterators to be used. Any number of iterators may be specified. The method collaboration logic follows that of either the Agent-Based Optimization or HOPSPACK codes and is currently under development. Table 4.4 summarizes the collaborative hybrid strategy inputs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid strategy</td>
<td>hybrid</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Collaborative hybrid</td>
<td>collaborative</td>
<td>none</td>
<td>Required group (1 of 3 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>List of methods</td>
<td>method_list</td>
<td>list of strings</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 4.4: Specification detail for collaborative hybrid strategies

4.5 Multistart Iteration Commands

The multi_start strategy must specify an iterator using method_pointer. This iterator is responsible for completing a series of iterative analyses from a set of different starting points. These starting points can be specified as follows: (1) using random_starts, for which the specified number of starting points are selected randomly within the variable bounds, (2) using starting_points, in which the starting values are provided in a list, or (3) using both random_starts and starting_points, for which the combined set of points will be used. In aggregate, at least one starting point must be specified. The most common example of a multi-start strategy is multi-start optimization, in which a series of optimizations are performed from different starting values for the design variables. This can be an effective approach for problems with multiple minima. Table 4.5 summarizes the multi-start strategy inputs.
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-start iteration strategy</td>
<td>multi_start</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Method pointer</td>
<td>method.pointer</td>
<td>string</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of random starting points</td>
<td>random_starts</td>
<td>integer</td>
<td>Optional group</td>
<td>no random starting points</td>
</tr>
<tr>
<td>Seed for random starting points</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>system-generated seed</td>
</tr>
<tr>
<td>List of user-specified starting points</td>
<td>starting_points</td>
<td>list of reals</td>
<td>Optional</td>
<td>no user-specified starting points</td>
</tr>
</tbody>
</table>

Table 4.5: Specification detail for multi-start strategies

4.6 Pareto Set Optimization Commands

The `pareto_set` strategy must specify an optimization or least squares calibration method using `method_pointer`. This minimizer is responsible for computing a set of optimal solutions from a set of response weightings (multi-objective weights or least squares term weights). These weightings can be specified as follows: (1) using `random_weight_sets`, in which case weightings are selected randomly within [0,1] bounds, (2) using `weight_sets`, in which the weighting sets are specified in a list, or (3) using both `random_weight_sets` and `weight_sets`, for which the combined set of weights will be used. In aggregate, at least one set of weights must be specified. The set of optimal solutions is called the "pareto set," which can provide valuable design trade-off information when there are competing objectives. Table 4.6 summarizes the pareto set strategy inputs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto set optimization strategy</td>
<td>pareto_set</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Optimization method pointer</td>
<td>method_pointer</td>
<td>string</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of random weighting sets</td>
<td>random_weight_sets</td>
<td>integer</td>
<td>Optional</td>
<td>no random weighting sets</td>
</tr>
<tr>
<td>Seed for random weighting sets</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>system-generated seed</td>
</tr>
<tr>
<td>List of user-specified weighting sets</td>
<td>weight_sets</td>
<td>list of reals</td>
<td>Optional</td>
<td>no user-specified weighting sets</td>
</tr>
</tbody>
</table>

Table 4.6: Specification detail for pareto set strategies

4.7 Single Method Commands

The single method strategy is the default if no strategy specification is included in a user input file. It may also be specified using the `single_method` keyword within a strategy specification. An optional `method_pointer` specification may be used to point to a particular method specification. If `method_pointer` is not used, then
the last method specification parsed will be used as the iterator. Table 4.7 summarizes the single method strategy inputs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single method strategy</td>
<td>single_method</td>
<td>string</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Method pointer</td>
<td>method_pointer</td>
<td>string</td>
<td>Optional</td>
<td>use of last method parsed</td>
</tr>
</tbody>
</table>

Table 4.7: Specification detail for single method strategies
Chapter 5

Method Commands

5.1 Method Description

The method section in a DAKOTA input file specifies the name and controls of an iterator. The terms "method" and "iterator" can be used interchangeably, although method often refers to an input specification whereas iterator usually refers to an object within the Iterator hierarchy. A method specification, then, is used to select an iterator from the iterator hierarchy, which includes optimization, uncertainty quantification, least squares, design of experiments, and parameter study iterators (see the Users Manual [Adams et al., 2010] for more information on these iterator branches). This iterator may be used alone or in combination with other iterators as dictated by the strategy specification (refer to Strategy Commands for strategy command syntax and to the Users Manual [Adams et al., 2010] for strategy algorithm descriptions).

Several examples follow. The first example shows a minimal specification for an optimization method.

```plaintext
method,
dot_sqp
```

This example uses all of the defaults for this method.

A more sophisticated example would be

```plaintext
method,
id_method = 'NLP1'
model_pointer = 'M1'
dot_sqp
  max_iterations = 50
  convergence_tolerance = 1e-4
  output verbose
  optimization_type minimize
```

This example demonstrates the use of identifiers and pointers (see Method Independent Controls) as well as some method independent and method dependent controls for the sequential quadratic programming (SQP) algorithm from the DOT library. The `max_iterations`, `convergence_tolerance`, and `output` settings are method independent controls, in that they are defined for a variety of methods (see DOT method independent controls for DOT usage of these controls). The `optimization_type` control is a method dependent control, in that it is only meaningful for DOT methods (see DOT method dependent controls).
The next example shows a specification for a least squares method.

```
method, optpp_g_newton
  max_iterations = 10
  convergence_tolerance = 1.e-8
  search_method trust_region
  gradient_tolerance = 1.e-6
```

Some of the same method independent controls are present along with a new set of method dependent controls (search_method and gradient_tolerance) which are only meaningful for OPT++ methods (see OPT++ method dependent controls).

The next example shows a specification for a nondeterministic method with several method dependent controls (refer to Nondeterministic sampling method).

```
method, sampling
  samples = 100 seed = 12345
  sample_type lhs
  response_levels = 1000. 500.
```

The last example shows a specification for a parameter study method where, again, each of the controls are method dependent (refer to Vector parameter study).

```
method, vector_parameter_study
  step_vector = 1. 1. 1.
  num_steps = 10
```

### 5.2 Method Specification

As alluded to in the examples above, the method specification has the following structure:

```
method, <method independent controls>
<method selection>
<method dependent controls>
```

where <method selection> is for example one of the following: dot_frcg, dot_mmfd, dot_bfgs, dot_slp, dot_sqp, conmin_frcg, conmin_mfd, npsol_sqp, nlssol_sqp, nlpqi_sqp, nl2sol, nonlinear_cg, optpp_cg, optpp_g_newton, optpp_fd_newton, optpp_pds, optpp_newton, coliny_cobyla, coliny_direct, coliny_pattern_search, coliny_solis_wets, coliny_ea, moga, soga, ncsu_direct, dl_solver, surrogate_based_local, surrogate_based_global, efficient_global, polynomial_chaos, stoch_collocation, sampling, importance, local_reliability, global_reliability, local_evidence, global_evidence, local_interval_est, global_interval_est, bayes_calib, dace, fsu_quasi_mc, fsu_cvt, psuade_moat, vector_parameter_study, list_parameter_study, centered_parameter_study, or multidim_parameter_study.
The method independent controls are those controls which are valid for a variety of methods. In some cases, these controls are abstractions which may have slightly different implementations from one method to the next. The method dependent controls are those controls which are only meaningful for a specific method or library. Referring to dakota.input.summary, the method independent controls are those controls defined externally from and prior to the method selection blocks. They are all optional. The method selection blocks are all required group specifications separated by logical OR’s. The method dependent controls are those controls defined within the method selection blocks. Defaults for method independent and method dependent controls are defined in DataMethod. The following sections provide additional detail on the method independent controls followed by the method selections and their corresponding method dependent controls.

5.3 Method Independent Controls

The method independent controls include a method identifier string, a model type specification with pointers to variables, interface, and responses specifications, a speculative gradient selection, an output verbosity control, maximum iteration and function evaluation limits, constraint and convergence tolerance specifications, a scaling selection, and a set of linear inequality and equality constraint specifications. While each of these controls is not valid for every method, the controls are valid for enough methods that it was reasonable to pull them out of the method dependent blocks and consolidate the specifications.

The method identifier string is supplied with id_method and is used to provide a unique identifier string for use with strategy specifications (refer to Strategy Description). It is appropriate to omit a method identifier string if only one method is included in the input file and single_method is the selected strategy (all other strategies require one or more method pointers), since the single method to use is unambiguous in this case.

The model pointer string is specified with model_pointer and is used to identify the model used to perform function evaluations for the method. If a model pointer string is specified and no corresponding id is available, DAKOTA will exit with an error message. If no model pointer string is specified, then the last model specification parsed will be used. If no model pointer string is specified and no model specification is provided by the user, then a default model specification is used (similar to the default strategy specification, see Strategy Description). This default model specification is of type single with no variables_pointer, interface_pointer, or responses_pointer (see Single Model Controls). It is appropriate to omit a model specification whenever the relationships are unambiguous due to the presence of single variables, interface, and responses specifications.

When performing gradient-based optimization in parallel, speculative gradients can be selected to address the load imbalance that can occur between gradient evaluation and line search phases. In a typical gradient-based optimization, the line search phase consists primarily of evaluating the objective function and any constraints at a trial point, and then testing the trial point for a sufficient decrease in the objective function value and/or constraint violation. If a sufficient decrease is not observed, then one or more additional trial points may be attempted sequentially. However, if the trial point is accepted then the line search phase is complete and the gradient evaluation phase begins. By speculating that the gradient information associated with a given line search trial point will be used later, additional coarse grained parallelism can be introduced by computing the gradient information (either by finite difference or analytically) in parallel, at the same time as the line search phase trial-point function values. This balances the total amount of computation to be performed at each design point and allows for efficient utilization of multiple processors. While the total amount of work performed will generally increase (since some speculative gradients will not be used when a trial point is rejected in the line search phase), the run time will usually decrease (since gradient evaluations needed at the start of each new optimization cycle were already performed in parallel during the line search phase). Refer to [Byrd et al., 1998] for additional details. The speculative specification is implemented for the gradient-based optimizers in the DOT, CONMIN, and OPT++ libraries, and it can be used with dakota numerical or analytic gradient selections in the responses specification.
Method Commands

(refer to Gradient Specification for information on these specifications). It should not be selected with vendor numerical gradients since vendor internal finite difference algorithms have not been modified for this purpose. In full-Newton approaches, the Hessian is also computed speculatively. NPSOL and NLSSOL do not support speculative gradients, as their gradient-based line search in user-supplied gradient mode (dakota numerical or analytic gradients) is a superior approach for load-balanced parallel execution.

Output verbosity control is specified with output followed by silent, quiet, verbose or debug. If there is no user specification for output verbosity, then the default setting is normal. This gives a total of five output levels to manage the volume of data that is returned to the user during the course of a study, ranging from full run annotation plus internal debug diagnostics (debug) to the bare minimum of output containing little more than the total number of simulations performed and the final solution (silent). Output verbosity is observed within the Iterator (algorithm verbosity), Model (synchronize/fd_gradients verbosity), Interface (map/synch verbosity), Approximation (global data fit coefficient reporting), and AnalysisCode (file operation reporting) class hierarchies; however, not all of these software components observe the full granularity of verbosity settings. Specific mappings are as follows:

- output silent (i.e., really quiet): silent iterators, silent model, silent interface, quiet approximation, quiet file operations
- output quiet: quiet iterators, quiet model, quiet interface, quiet approximation, quiet file operations
- output normal: normal iterators, normal model, normal interface, quiet approximation, quiet file operations
- output verbose: verbose iterators, normal model, verbose interface, verbose approximation, verbose file operations
- output debug (i.e., really verbose): debug iterators, normal model, debug interface, verbose approximation, verbose file operations

Note that iterators and interfaces utilize the full granularity in verbosity, whereas models, approximations, and file operations do not. With respect to iterator verbosity, different iterators implement this control in slightly different ways (as described below in the method independent controls descriptions for each iterator), however the meaning is consistent. For models, interfaces, approximations, and file operations, quiet suppresses parameter and response set reporting and silent further suppresses function evaluation headers and scheduling output. Similarly, verbose adds file management, approximation evaluation, and global approximation coefficient details, and debug further adds diagnostics from nonblocking schedulers.

The constraint_tolerance specification determines the maximum allowable value of infeasibility that any constraint in an optimization problem may possess and still be considered to be satisfied. It is specified as a positive real value. If a constraint function is greater than this value then it is considered to be violated by the optimization algorithm. This specification gives some control over how tightly the constraints will be satisfied at convergence of the algorithm. However, if the value is set too small the algorithm may terminate with one or more constraints being violated. This specification is currently meaningful for the NPSOL, NLSSOL, DOT and CONMIN constrained optimizers (refer to DOT method independent controls and NPSOL method independent controls).

The convergence_tolerance specification provides a real value for controlling the termination of iteration. In most cases, it is a relative convergence tolerance for the objective function; i.e., if the change in the objective function between successive iterations divided by the previous objective function is less than the amount specified by convergence_tolerance, then this convergence criterion is satisfied on the current iteration. Since no progress may be made on one iteration followed by significant progress on a subsequent iteration, some libraries
require that the convergence tolerance be satisfied on two or more consecutive iterations prior to termination of iteration. This control is used with optimization and least squares iterators (DOT, CONMIN, NPSOL, NLSSOL, OPT++, and Coliny) and is not used within the uncertainty quantification, design of experiments, or parameter study iterator branches. Refer to DOT method independent controls, NPSOL method independent controls, OPT++ method independent controls, and Coliny method independent controls for specific interpretations of the convergence_tolerance specification.

The max_iterations and max_function_evaluations controls provide integer limits for the maximum number of iterations and maximum number of function evaluations, respectively. The difference between an iteration and a function evaluation is that a function evaluation involves a single parameter to response mapping through an interface, whereas an iteration involves a complete cycle of computation within the iterator. Thus, an iteration generally involves multiple function evaluations (e.g., an iteration contains descent direction and line search computations in gradient-based optimization, population and multiple offset evaluations in nongradient-based optimization, etc.). The max_function_evaluations control is not currently used within the uncertainty quantification, design of experiments, and parameter study iterator branches, and in the case of gradient-based methods, does not currently capture function evaluations that occur as part of the method_source dakota finite difference routine (since these additional evaluations are intentionally isolated from the iterators).

The final_solutions controls the number of final solutions returned by the iterator as the best solutions. For most optimizers, this is one, but some optimizers can produce multiple solutions (e.g. genetic algorithms). In the case of analyzers such as sampling methods, if one specifies 100 samples (for example) but also specifies final_solutions = 5, the five best solutions (in order of lowest response function value) are returned. When using a hybrid strategy, the number of final solutions dictates how many solutions are passed from one method to another.

Continuous design variable, function, and constraint scaling can be turned on for optimizers and least squares minimizers by providing the scaling keyword. Discrete variable scaling is not supported. When scaling is enabled, variables, functions, gradients, Hessians, etc., are transformed such that the optimizer iterates in scaled variable space, whereas evaluations of the computational model as specified in the interface are performed on the original problem scale. Therefore using scaling does not require rewriting the interface to the simulation code. The user may specify no, one, or a vector of scaling type strings through each of the scale_types (see Variables Commands); objective_function_scale_types, least_squares_term_scale_types, nonlinear_inequality_scale_types, nonlinear_equality_scale_types (see Function Specification); linear_inequality_scale_types, and linear_equality_scale_types (see Method Independent Controls below) specifications. Valid options for types include ‘none’ (default), ‘value’, ‘auto’, or ‘log’, for no, characteristic value, automatic, or logarithmic scaling, respectively, although not all types are valid for scaling all entities (see the references for details). If a single string is specified using any of these keywords it will apply to each component of the relevant vector, e.g., scale_types = ‘value’ will enable characteristic value scaling for each continuous design variable. The user may specify no, one, or a vector of nonzero characteristic scale values through each of the scales (see Variables Commands); objective_function_scales, least_squares_term_scales, nonlinear_inequality_scales, nonlinear_equality_scales (see Function Specification); linear_inequality_scales, and linear_equality_scales (see Method Independent Controls below) specifications. These values are ignored for scaling type ‘none’, required for ‘value’, and optional for ‘auto’ and ‘log’. If a single value is specified using any of these keywords it will apply to each component of the relevant vector, e.g., scales = 3.0 will apply a characteristic scaling value of 3.0 to each continuous design variable. When the scaling keyword is omitted, all _scale_types and _scales specifications are ignored in the method, variables, and responses sections.

When scaling is enabled, the following procedures determine the transformations used to scale each component

of a variables or response vector. A warning is issued if scaling would result in division by a value smaller in magnitude than $1.0 \times 10^{-10} \times \text{DBL_MIN}$. User-provided values violating this lower bound are accepted unaltered, whereas for automatically calculated scaling, the lower bound is enforced.

- **None** (‘none’): no scaling performed (**_scales** ignored) on this component
- **Characteristic value** (‘value’): the corresponding quantity is scaled by the (required) characteristic value provided in the **_scales** specification. If the scale value is negative, the sense of inequalities are changed accordingly.
- **Automatic** (‘auto’): First, any characteristic values from the optional **_scales** specification are applied. Then, automatic scaling will be attempted according to the following scheme:
  - two-sided bounds scaled into the interval $[0,1]$;
  - one-sided bound or targets are scaled by the characteristic value, moving the bound or target to 1 and changing the sense of inequalities where necessary;
  - no bounds or targets: no automatic scaling possible, therefore no scaling for this component

Automatic scaling is not available for objective functions nor least squares terms since they lack bound constraints. Further, when automatically scaled, linear constraints are scaled by characteristic values only, not affinely scaled into $[0,1]$.

- **Logarithmic** (‘log’): First, any characteristic values from the optional **_scales** specification are applied. Then, logarithm base 10 scaling is applied. Logarithmic scaling is not available for linear constraints. Further, when continuous design variables are log scaled, linear constraints are not allowed.

**Table 5.1** provides the specification detail for the method independent controls involving identifiers, pointers, tolerances, limits, output verbosity, speculative gradients, and scaling.

Linear inequality constraints can be supplied with the **linear_inequality_constraint_matrix**, **linear_inequality_lower_bounds**, and **linear_inequality_upper_bounds** specifications, and linear equality constraints can be supplied with the **linear_equality_constraint_matrix** and **linear_equality_targets** specifications. In the inequality case, the constraint matrix provides coefficients for the variables and the lower and upper bounds provide constraint limits for the following two-sided formulation:

$$a_l \leq Ax \leq a_u$$

As with nonlinear inequality constraints (see **Objective and constraint functions (optimization data set)**), the default linear inequality constraint bounds are selected so that one-sided inequalities of the form

$$Ax \leq 0.0$$

result when there are no user bounds specifications (this provides backwards compatibility with previous DAKOTA versions). In a user bounds specification, any upper bound values greater than $+\text{bigRealBoundSize} (1.0 \times 10^{30}, \text{as defined in Minimizer})$ are treated as $+\infty$ and any lower bound values less than $-\text{bigRealBoundSize}$ are treated as $-\infty$. This feature is commonly used to drop one of the bounds in order to specify a 1-sided constraint (just as the default lower bounds drop out since $-\text{DBL_MAX} < -\text{bigRealBoundSize}$). In the equality case, the constraint matrix again provides coefficients for the variables and the targets provide the equality constraint right hand sides:

$$Ax = a_t$$
### Method Independent Controls

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method set identifier</td>
<td>id_method</td>
<td>string</td>
<td>Optional</td>
<td>strategy use of last method parsed</td>
</tr>
<tr>
<td>Model pointer</td>
<td>model_pointer</td>
<td>string</td>
<td>Optional</td>
<td>method use of last model parsed (or use of default model if none parsed)</td>
</tr>
<tr>
<td>Speculative gradients and Hessians</td>
<td>speculative</td>
<td>none</td>
<td>Optional</td>
<td>no speculation</td>
</tr>
<tr>
<td>Output verbosity</td>
<td>output</td>
<td>silent</td>
<td>quiet</td>
<td>verbose</td>
</tr>
<tr>
<td>Maximum iterations</td>
<td>max_iterations</td>
<td>integer</td>
<td>Optional</td>
<td>100 (exceptions: fsu_cvt/local_reliability: 25, global_{reliability,interval_est,evidence}/efficient_global: 25*n)</td>
</tr>
<tr>
<td>Maximum function evaluations</td>
<td>max_function_evaluations</td>
<td>integer</td>
<td>Optional</td>
<td>1000</td>
</tr>
<tr>
<td>Constraint tolerance</td>
<td>constraint_tolerance</td>
<td>real</td>
<td>Optional</td>
<td>Library default</td>
</tr>
<tr>
<td>Convergence tolerance</td>
<td>convergence_tolerance</td>
<td>real</td>
<td>Optional</td>
<td>1.e-4</td>
</tr>
<tr>
<td>Final solutions</td>
<td>final_solutions</td>
<td>integer</td>
<td>Optional</td>
<td>1</td>
</tr>
<tr>
<td>Scaling flag</td>
<td>scaling</td>
<td>none</td>
<td>Optional</td>
<td>no scaling</td>
</tr>
</tbody>
</table>

Table 5.1: Specification detail for the method independent controls: identifiers, pointers, tolerances, limits, output verbosity, speculative gradients, and scaling
and the defaults for the equality constraint targets enforce a value of 0. for each constraint

\[ Ax = 0.0 \]

Currently, DOT, CONMIN, NPSOL, NLSSOL, and OPT++ all support specialized handling of linear constraints (either directly through the algorithm itself or indirectly through the DAKOTA wrapper). Coliny optimizers will support linear constraints in future releases. Linear constraints need not be computed by the user’s interface on every function evaluation; rather the coefficients, bounds, and targets of the linear constraints can be provided at start up, allowing the optimizers to track the linear constraints internally. It is important to recognize that linear constraints are those constraints that are linear in the design variables, e.g.:

\[
\begin{align*}
0.0 & \leq 3x_1 - 4x_2 + 2x_3 \leq 15.0 \\
x_1 + x_2 + x_3 & \geq 2.0 \\
x_1 + x_2 - x_3 & = 1.0
\end{align*}
\]

which is not to be confused with something like

\[ s(X) - s_{fail} \leq 0.0 \]

where the constraint is linear in a response quantity, but may be a nonlinear implicit function of the design variables. For the three linear constraints above, the specification would appear as:

```plaintext
linear_inequality_constraint_matrix = 3.0 -4.0 2.0
                                           1.0 1.0 1.0
linear_inequality_lower_bounds = 0.0 2.0
linear_inequality_upper_bounds = 15.0 1.e+50
linear_equality_constraint_matrix = 1.0 1.0 -1.0
linear_equality_targets = 1.0
```

where the 1.e+50 is a dummy upper bound value which defines a 1-sided inequality since it is greater than `bigRealBoundSize`. The constraint matrix specifications list the coefficients of the first constraint followed by the coefficients of the second constraint, and so on. They are divided into individual constraints based on the number of design variables, and can be broken onto multiple lines for readability as shown above.

The `linear_inequality_scale_types` and `linear_equality_scale_types` specifications provide strings specifying the scaling type for each linear inequality or equality constraint, respectively, in methods that support scaling, when scaling is enabled (see Method Independent Controls for details). Each entry in `linear_\*_scale_types` may be selected from ‘none’, ‘value’, or ‘auto’ to select no, characteristic value, or automatic scaling, respectively. If a single string is specified it will apply to each constraint component. Each entry in `linear_inequality_scales` or `linear_equality_scales` may be a user-specified nonzero characteristic value to be used in scaling each constraint. These values are ignored for scaling type ‘none’, required for ‘value’, and optional for ‘auto’. If a single real value is specified it will apply to all components of the constraint. Scaling for linear constraints is applied after any continuous variable scaling. For example, for variable scaling on continuous design variables \( x \):

\[
\tilde{x}^j = \frac{x^j - x^j_Q}{x^j_M}
\]

we have the following system for linear inequality constraints

\[ a_L \leq A_i x \leq a_U \]
and user-specified or automatically computed scaling multipliers are applied to this final transformed system, which accounts for continuous design variable scaling. When automatic scaling is in use for linear constraints they are linearly scaled by a computed characteristic value, but not affinely to [0,1].

Table 5.2 provides the specification detail for the method independent controls involving linear constraints.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear inequality coefficient matrix</td>
<td>linear_inequality_constraint_matrix</td>
<td>list of reals</td>
<td>Optional</td>
<td>no linear inequality constraints</td>
</tr>
<tr>
<td>Linear inequality lower bounds</td>
<td>linear_inequality_lower_bounds</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = -DBL_MAX</td>
</tr>
<tr>
<td>Linear inequality upper bounds</td>
<td>linear_inequality_upper_bounds</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 0.</td>
</tr>
<tr>
<td>Linear inequality scaling types</td>
<td>linear_inequality_scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
</tr>
<tr>
<td>Linear inequality scales</td>
<td>linear_inequality_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 1. (no scaling)</td>
</tr>
<tr>
<td>Linear equality coefficient matrix</td>
<td>linear_equality_constraint_matrix</td>
<td>list of reals</td>
<td>Optional</td>
<td>no linear equality constraints</td>
</tr>
<tr>
<td>Linear equality targets</td>
<td>linear_equality_targets</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 0.</td>
</tr>
<tr>
<td>Linear equality scaling types</td>
<td>linear_equality_scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
</tr>
<tr>
<td>Linear equality scales</td>
<td>linear_equality_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 1. (no scaling)</td>
</tr>
</tbody>
</table>

Table 5.2: Specification detail for the method independent controls: linear inequality and equality constraints

5.4 Optimization Methods

The DAKOTA project started as an toolbox for optimization methods, and has accumulated a broad variety of gradient-based and nongradient-based optimizers from the DOT, NPSOL, NLPQL, CONMIN, OPT++, APPS, COLINY, NCSU, and JEGA packages. These capabilities are described below.
5.4.1 DOT Methods

The DOT library [Vanderplaats Research and Development, 1995] contains nonlinear programming optimizers, specifically the Broyden-Fletcher-Goldfarb-Shanno (DAKOTA’s dot_bfgs method) and Fletcher-Reeves conjugate gradient (DAKOTA’s dot_frcg method) methods for unconstrained optimization, and the modified method of feasible directions (DAKOTA’s dot_mmfd method), sequential linear programming (DAKOTA’s dot_slp method), and sequential quadratic programming (DAKOTA’s dot_sqp method) methods for constrained optimization. DAKOTA provides access to the DOT library through the DOTOptimizer class.

**DOT method independent controls** The method independent controls for max_iterations and max_function_evaluations limit the number of major iterations and the number of function evaluations that can be performed during a DOT optimization. The convergence_tolerance control defines the threshold value on relative change in the objective function that indicates convergence. This convergence criterion must be satisfied for two consecutive iterations before DOT will terminate. The constraint_tolerance specification defines how tightly constraint functions are to be satisfied at convergence. The default value for DOT constrained optimizers is 0.003. Extremely small values for constraint_tolerance may not be attainable. The output verbosity specification controls the amount of information generated by DOT: the silent and quiet settings result in header information, final results, and objective function, constraint, and parameter information on each iteration; whereas the verbose and debug settings add additional information on gradients, search direction, one-dimensional search results, and parameter scaling factors. DOT contains no parallel algorithms which can directly take advantage of concurrent evaluations. However, if numerical_gradients with method_source dakota is specified, then the finite difference function evaluations can be performed concurrently (using any of the parallel modes described in the Users Manual [Adams et al., 2010]). In addition, if speculative is specified, then gradients (dakota numerical or analytic gradients) will be computed on each line search evaluation in order to balance the load and lower the total run time in parallel optimization studies. Lastly, specialized handling of linear constraints is supported with DOT; linear constraint coefficients, bounds, and targets can be provided to DOT at start-up and tracked internally. Specification detail for these method independent controls is provided in Tables 5.1 through 5.2.

**DOT method dependent controls** DOT’s only method dependent control is optimization_type which may be either minimize or maximize. DAKOTA provides the only set of methods within DAKOTA which support this control; to convert a maximization problem into the minimization formulation assumed by other methods, simply change the sign on the objective function (i.e., multiply by -1). Table 5.3 provides the specification detail for the DOT methods and their method dependent controls.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization type</td>
<td>optimization_type</td>
<td>minimize</td>
<td>maximize</td>
<td>Optional group</td>
</tr>
</tbody>
</table>

Table 5.3: Specification detail for the DOT methods

5.4.2 NPSOL Method

The NPSOL library [Gill et al., 1986] contains a sequential quadratic programming (SQP) implementation (the npsol_sqp method). SQP is a nonlinear programming optimizer for constrained minimization. DAKOTA provides access to the NPSOL library through the NPSOLOptimizer class.
**NPSOL method independent controls**  The method independent controls for `max_iterations` and `max_function_evaluations` limit the number of major SQP iterations and the number of function evaluations that can be performed during an NPSOL optimization. The `convergence_tolerance` control defines NPSOL's internal optimality tolerance which is used in evaluating if an iterate satisfies the first-order Kuhn-Tucker conditions for a minimum. The magnitude of `convergence_tolerance` approximately specifies the number of significant digits of accuracy desired in the final objective function (e.g., `convergence_tolerance = 1.e-6` will result in approximately six digits of accuracy in the final objective function). The `constraint_tolerance` control defines how tightly the constraint functions are satisfied at convergence. The default value is dependent upon the machine precision of the platform in use, but is typically on the order of `1.e-8` for double precision computations. Extremely small values for `constraint_tolerance` may not be attainable.

The `output` verbosity setting controls the amount of information generated at each major SQP iteration: the `silent` and `quiet` settings result in only one line of diagnostic output for each major iteration and print the final optimization solution, whereas the `verbose` and `debug` settings add additional information on the objective function, constraints, and variables at each major iteration.

NPSOL is not a parallel algorithm and cannot directly take advantage of concurrent evaluations. However, if `numerical_gradients` with `method_source dakota` is specified, then the finite difference function evaluations can be performed concurrently (using any of the parallel modes described in the Users Manual [Adams et al., 2010]). An important related observation is the fact that NPSOL uses two different line searches depending on how gradients are computed. For either `analytic_gradients` or `numerical_gradients` with `method_source dakota`, NPSOL is placed in user-supplied gradient mode (NPSOL's "Derivative Level" is set to 3) and it uses a gradient-based line search (the assumption is that user-supplied gradients are inexpensive). On the other hand, if `numerical_gradients` are selected with `method_source vendor`, then NPSOL is computing finite differences internally and it will use a value-based line search (the assumption is that finite differencing on each line search evaluation is too expensive). The ramifications of this are: (1) performance will vary between `method_source dakota` and `method_source vendor` for `numerical_gradients`, and (2) gradient speculation is unnecessary when performing optimization in parallel since the gradient-based line search in user-supplied gradient mode is already load balanced for parallel execution. Therefore, a speculative specification will be ignored by NPSOL, and optimization with numerical gradients should select `method_source dakota` for load balanced parallel operation and `method_source vendor` for efficient serial operation.

Lastly, NPSOL supports specialized handling of linear inequality and equality constraints. By specifying the coefficients and bounds of the linear inequality constraints and the coefficients and targets of the linear equality constraints, this information can be provided to NPSOL at initialization and tracked internally, removing the need for the user to provide the values of the linear constraints on every function evaluation. Refer to **Method Independent Controls** for additional information and to Tables 5.1 through 5.2 for method independent control specification detail.

**NPSOL method dependent controls**  NPSOL's method dependent controls are `verify_level`, `function_precision`, and `linesearch_tolerance`. The `verify_level` control instructs NPSOL to perform finite difference verifications on user-supplied gradient components. The `function_precision` control provides NPSOL an estimate of the accuracy to which the problem functions can be computed. This is used to prevent NPSOL from trying to distinguish between function values that differ by less than the inherent error in the calculation. And the `linesearch_tolerance` setting controls the accuracy of the line search. The smaller the value (between 0 and 1), the more accurately NPSOL will attempt to compute a precise minimum along the search direction. Table 5.4 provides the specification detail for the NPSOL SQP method and its method dependent controls.
5.4.3 NLPQL Methods

The NLPQL library is a commercially-licensed library containing a sequential quadratic programming (SQP) optimizer, specified as DAKOTA’s nlpql_sqp method, for constrained optimization. The particular implementation used is NLPQLP [Schittkowski, 2004], a variant with distributed and non-monotone line search. DAKOTA provides access to the NLPQL library through the NLPQLPOptimizer class.

**NLPQL method independent controls**  The method independent controls for maximum iterations and output verbosity are mapped to NLPQL controls MAXIT and IPRINT, respectively. The maximum number of function evaluations is enforced within the NLPQLPOptimizer class.

**NLPQL method dependent controls**  NLPQL does not currently support any method dependent controls.

5.4.4 CONMIN Methods

The CONMIN library [Vanderplaats, 1973] is a public domain library of nonlinear programming optimizers, specifically the Fletcher-Reeves conjugate gradient (DAKOTA’s conmin_frcg method) method for unconstrained optimization, and the method of feasible directions (DAKOTA’s conmin_mfd method) for constrained optimization. As CONMIN was a predecessor to the DOT commercial library, the algorithm controls are very similar. DAKOTA provides access to the CONMIN library through the CONMINOptimizer class.

**CONMIN method independent controls**  The interpretations of the method independent controls for CONMIN are essentially identical to those for DOT. Therefore, the discussion in DOT method independent controls is relevant for CONMIN.

**CONMIN method dependent controls**  CONMIN does not currently support any method dependent controls.

5.4.5 OPT++ Methods

The OPT++ library [Meza et al., 2007] contains primarily gradient-based nonlinear programming optimizers for unconstrained, bound-constrained, and nonlinearly constrained minimization: Polak-Ribiere conjugate gradient (DAKOTA’s optpp_cg method), quasi-Newton (DAKOTA’s optpp_q_newton method), finite difference Newton (DAKOTA’s optpp_fd_newton method), and full Newton (DAKOTA’s optpp_newton method). The conjugate gradient method is strictly unconstrained, and each of the Newton-based methods are automatically
bound to the appropriate OPT++ algorithm based on the user constraint specification (unconstrained, bound-constrained, or generally-constrained). In the generally-constrained case, the Newton methods use a nonlinear interior-point approach to manage the constraints. The library also contains a direct search algorithm, PDS (parallel direct search, DAKOTA’s optpp_pds method), which supports bound constraints. DAKOTA provides access to the OPT++ library through the SNLLOptimizer class, where “SNLL” denotes Sandia National Laboratories - Livermore.

**OPT++ method independent controls** The method independent controls for max_iterations and max_function_evaluations limit the number of major iterations and the number of function evaluations that can be performed during an OPT++ optimization. The convergence_tolerance control defines the threshold value on relative change in the objective function that indicates convergence. The output verbosity specification controls the amount of information generated from OPT++ executions: the debug setting turns on OPT++’s internal debug mode and also generates additional debugging information from DAKOTA’s SNLLOptimizer wrapper class. OPT++’s gradient-based methods are not parallel algorithms and cannot directly take advantage of concurrent function evaluations. However, if numerical_gradients with method_source dakota is specified, a parallel DAKOTA configuration can utilize concurrent evaluations for the finite difference gradient computations. OPT++’s nongradient-based PDS method can directly exploit asynchronous evaluations; however, this capability has not yet been implemented in the SNLLOptimizer class.

The speculative specification enables speculative computation of gradient and/or Hessian information, where applicable, for parallel optimization studies. By speculating that the derivative information at the current point will be used later, the complete data set (all available gradient/Hessian information) can be computed on every function evaluation. While some of these computations will be wasted, the positive effects are a consistent parallel load balance and usually shorter wall clock time. The speculative specification is applicable only when parallelism in the gradient calculations can be exploited by DAKOTA (it will be ignored for vendor numerical gradients).

Lastly, linear constraint specifications are supported by each of the Newton methods (optpp_newton, optpp_q_newton, optpp_fd_newton, and optpp_g_newton); whereas optpp_cg must be unconstrained and optpp_pds can be, at most, bound-constrained. Specification detail for the method independent controls is provided in Tables 5.1 through 5.2.

**OPT++ method dependent controls** OPT++’s method dependent controls are max_step, gradient_tolerance, search_method, merit_function, central_path, steplength_to_boundary, centering_parameter, and search_scheme_size. The max_step control specifies the maximum step that can be taken when computing a change in the current design point (e.g., limiting the Newton step computed from current gradient and Hessian information). It is equivalent to a move limit or a maximum trust region size. The gradient_tolerance control defines the threshold value on the L2 norm of the objective function gradient that indicates convergence to an unconstrained minimum (no active constraints). The gradient_tolerance control is defined for all gradient-based optimizers.

max_step and gradient_tolerance are the only method dependent controls for the OPT++ conjugate gradient method. Table 5.5 covers this specification.

The search_method control is defined for all Newton-based optimizers and is used to select between trust_region, gradient_based_line_search, and value_based_line_search methods. The gradient_based_line_search option uses the line search method proposed by [More and Thuente, 1994]. This option satisfies sufficient decrease and curvature conditions; whereas, value_base_line_search only satisfies the sufficient decrease condition. At each line search iteration, the gradient_based_line_search method computes the function and gradient at the trial point. Consequently, given expensive function evaluations,
Opt++ conjugate gradient method

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
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<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Maximum step size</td>
<td>max_step</td>
<td>real</td>
<td>Optional</td>
<td>1000.</td>
</tr>
<tr>
<td>Gradient tolerance</td>
<td>gradient_-</td>
<td>real</td>
<td>Optional</td>
<td>1.e-4</td>
</tr>
</tbody>
</table>

Table 5.5: Specification detail for the OPT++ conjugate gradient method

The value_based_line_search method is preferred to the gradient_based_line_search method. Each of these Newton methods additionally supports the tr_pds selection for unconstrained problems. This option performs a robust trust region search using pattern search techniques. Use of a line search is the default for bound-constrained and generally-constrained problems, and use of a trust_region search method is the default for unconstrained problems.

The merit_function, central_path, steplength_to_boundary, and centering_parameter selections are additional specifications that are defined for the solution of generally-constrained problems with nonlinear interior-point algorithms. A merit_function is a function in constrained optimization that attempts to provide joint progress toward reducing the objective function and satisfying the constraints. Valid string inputs are "el_bakry", "argaez_tapia", or "van_shanno", where user input is not case sensitive in this case. Details for these selections are as follows:

- The "el_bakry" merit function is the L2-norm of the first order optimality conditions for the nonlinear programming problem. The cost per linesearch iteration is n+1 function evaluations. For more information, see [El-Bakry et al., 1996].

- The "argaez_tapia" merit function can be classified as a modified augmented Lagrangian function. The augmented Lagrangian is modified by adding to its penalty term a potential reduction function to handle the perturbed complementarity condition. The cost per linesearch iteration is one function evaluation. For more information, see [Tapia and Argaez].

- The "van_shanno" merit function can be classified as a penalty function for the logarithmic barrier formulation of the nonlinear programming problem. The cost per linesearch iteration is one function evaluation. For more information see [Vanderbei and Shanno, 1999].

If the function evaluation is expensive or noisy, set the merit_function to "argaez_tapia" or "van_shanno".

The central_path specification represents a measure of proximity to the central path and specifies an update strategy for the perturbation parameter mu. Refer to [Argaez et al., 2002] for a detailed discussion on proximity measures to the central region. Valid options are, again, "el_bakry", "argaez_tapia", or "van_shanno", where user input is not case sensitive. The default value for central_path is the value of merit_function (either user-selected or default). The steplength_to_boundary specification is a parameter (between 0 and 1) that controls how close to the boundary of the feasible region the algorithm is allowed to move. A value of 1 means that the algorithm is allowed to take steps that may reach the boundary of the feasible region. If the user wishes to maintain strict feasibility of the design parameters this value should be less than 1. Default values are .8, .99995, and .95 for the "el_bakry", "argaez_tapia", and "van_shanno" merit functions, respectively. The centering_-parameter specification is a parameter (between 0 and 1) that controls how closely the algorithm should follow the "central path". See [Wright] for the definition of central path. The larger the value, the more closely the
5.4 Optimization Methods

algorithm follows the central path, which results in small steps. A value of 0 indicates that the algorithm will take a pure Newton step. Default values are .2, .2, and .1 for the "el_bakry", "argaez_tapia", and "van_shanno" merit functions, respectively.

Table 5.6 provides the details for the Newton-based methods.

<table>
<thead>
<tr>
<th>Description</th>
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<th>Associated Data</th>
<th>Status</th>
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<td>optpp_fd_newton</td>
<td>optpp_newton</td>
<td>none</td>
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<td>methods</td>
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<td></td>
<td></td>
<td>N/A</td>
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<td>value_based_line_search</td>
<td>gradient_based_line_search</td>
<td>trust_region</td>
<td>tr_pds</td>
</tr>
<tr>
<td>Maximum step size</td>
<td>max_step</td>
<td>real</td>
<td>Optional</td>
<td>1000.</td>
</tr>
<tr>
<td>Gradient tolerance</td>
<td>gradient_tolerance</td>
<td>real</td>
<td>Optional</td>
<td>1.e-4</td>
</tr>
<tr>
<td>Merit function</td>
<td>merit_function</td>
<td>string</td>
<td>Optional</td>
<td>&quot;argaez_tapia&quot;</td>
</tr>
<tr>
<td>Central path</td>
<td>central_path</td>
<td>string</td>
<td>Optional</td>
<td>value of merit_function</td>
</tr>
<tr>
<td>Steplength to boundary</td>
<td>steplength_to_boundary</td>
<td>real</td>
<td>Optional</td>
<td>Merit function dependent: 0.8 (el_bakry), 0.99995 (argaez_tapia), 0.95 (van_shanno)</td>
</tr>
<tr>
<td>Centering parameter</td>
<td>centering_parameter</td>
<td>real</td>
<td>Optional</td>
<td>Merit function dependent: 0.2 (el_bakry), 0.2 (argaez_tapia), 0.1 (van_shanno)</td>
</tr>
</tbody>
</table>

Table 5.6: Specification detail for OPT++ Newton-based optimization methods

The search_scheme_size is defined for the PDS method to specify the number of points to be used in the direct search template. PDS does not support parallelism at this time due to current limitations in the OPT++ interface. Table 5.7 provides the detail for the parallel direct search method.
### Table 5.7: Specification detail for the OPT++ PDS method

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
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<td>OPT++ parallel direct search method</td>
<td>optpp_pds</td>
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<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Search scheme size</td>
<td>search_scheme_size</td>
<td>integer</td>
<td>Optional</td>
<td>32</td>
</tr>
</tbody>
</table>

5.4.6 Asynchronous Parallel Pattern Search (APPS)

The asynchronous parallel pattern search (APPS) algorithm [Gray and Kolda, 2006] is a fully asynchronous pattern search technique in that the search along each offset direction continues without waiting for searches along other directions to finish. It is now made available in DAKOTA through the HOPSPACK software [Plantenga, 2009]. It can handle unconstrained problems as well as those with bound constraints, linear constraints, and general nonlinear constraints. HOPSPACK is available to the public under the GNU LGPL and the source code is included with DAKOTA. HOPSPACK-specific software documentation is available from https://software.sandia.gov/trac/hopspack.

#### APPS method independent controls

The only method independent controls that are currently mapped to APPS are `max_function_evaluations`, `constraint_tolerance`, and the output verbosity control. We note that while APPS treats the constraint tolerance separately for linear and nonlinear constraints, we apply the same value to both if the user specifies `constraint_tolerance`. The APPS internal "display" level is mapped to the DAKOTA `debug`, `verbose`, `normal`, `quiet`, and `silent` settings as follows:

- **DAKOTA "debug":** display final solution, all input parameters, variable and constraint info, trial points, search directions, and execution details
- **DAKOTA "verbose":** display final solution, all input parameters, variable and constraint info, and trial points
- **DAKOTA "normal":** display final solution, all input parameters, variable and constraint summaries, and new best points
- **DAKOTA "quiet":** display final solution and all input parameters
- **DAKOTA "silent":** display final solution

APPS exploits parallelism through the use of DAKOTA’s concurrent function evaluations. The variant of the algorithm that is currently exposed, however, limits the amount of concurrency that can be exploited. In particular, APPS can leverage an evaluation concurrency level of at most twice the number of variables. More options that allow for greater evaluation concurrency may be exposed in future releases.

#### APPS method dependent controls

The APPS method is invoked using a `asynch_pattern_search` group specification. Some of the method dependent controls are similar to the Coliny controls for `coliny_pattern_search` described in Pattern Search. In particular, APPS supports the following step length control parameters:

- **`initial_delta`:** the initial step length, must be positive
5.4 Optimization Methods

- **threshold_delta**: step length used to determine convergence, must be greater than or equal to 4.4e-16
- **contraction_factor**: amount by which step length is rescaled after unsuccessful iterates, must be strictly between 0 and 1

When the solution to the optimization problem is known, the user may specify a value for **solution_target** as a termination criteria. APPS will terminate when the function value falls below **solution_target**.

Currently, APPS only supports coordinate bases with a total of $2n$ function evaluations in the pattern, and these patterns may only contract. The **synchronization** specification can be used to specify the use of either **blocking** or **nonblocking** schedulers for APPS. The **blocking** option causes APPS to behave as a synchronous algorithm. The **nonblocking** option is not available when Dakota is used in message-passing mode.

APPS solves nonlinearly constrained problems by solving a sequence of linearly constrained merit function-base subproblems. There are several exact and smoothed exact penalty functions that can be specified with the **merit_function** control. The options are as follows:

- **merit_max**: based on $\ell_\infty$ norm
- **merit_max_smooth**: based on smoothed $\ell_\infty$ norm
- **merit1**: based on $\ell_1$ norm
- **merit1_smooth**: based on smoothed $\ell_1$ norm
- **merit2**: based on $\ell_2$ norm
- **merit2_smooth**: based on smoothed $\ell_2$ norm
- **merit2_squared**: based on $\ell_2^2$ norm

The user can also specify the following:

- **constraint_penalty**: the initial multiplier for the penalty function, must be nonnegative
- **smoothing_parameter**: initial smoothing value for smoothed penalty functions, must be between 0 and 1 (inclusive)

Table 5.8 summarizes the APPS specification.

### 5.4.7 Coliny Methods

Coliny is a collection of nongradient-based optimizers that support the Common Optimization Library INterface (COLIN). Coliny optimizers currently include coliny_cobyla, coliny_direct, coliny_ea, coliny_pattern_search and coliny_solis_wets. Additional Coliny information is available from [https://software.sandia.gov/trac/acro](https://software.sandia.gov/trac/acro).

Coliny solvers now support bound constraints and general nonlinear constraints. Supported nonlinear constraints include both equality and two-sided inequality constraints. Coliny solvers do not yet support linear constraints. Most Coliny optimizers treat constraints with a simple penalty scheme that adds **constraint_penalty** times the sum of squares of the constraint violations to the objective function. Specific exceptions to this method for handling constraint violations are noted below. (The default value of **constraint_penalty** is 1000.0, except for methods that dynamically adapt their constraint penalty, for which the default value is 1.0.)
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
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<td>APPS method</td>
<td>asynch_-pattern_-search</td>
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<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Initial offset value</td>
<td>initial_-delta</td>
<td>real</td>
<td>Optional</td>
<td>1.0</td>
</tr>
<tr>
<td>Threshold for offset values</td>
<td>threshold_-delta</td>
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<td>Optional</td>
<td>0.01</td>
</tr>
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<td>Pattern contraction factor</td>
<td>contraction_-factor</td>
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<td>Optional</td>
<td>0.5</td>
</tr>
<tr>
<td>Solution target</td>
<td>solution_-target</td>
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<td>Optional</td>
<td>not used</td>
</tr>
<tr>
<td>Evaluation synchronization</td>
<td>synchronization</td>
<td>blocking</td>
<td>nonblocking</td>
<td>Optional</td>
</tr>
<tr>
<td>Merit function</td>
<td>merit_-function</td>
<td>merit_max</td>
<td>merit_max_-smooth</td>
<td>merit1</td>
</tr>
<tr>
<td>Constraint penalty</td>
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<td>Optional</td>
<td>1.0</td>
</tr>
<tr>
<td>Smoothing factor</td>
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<td>Optional</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 5.8: Specification detail for the APPS method
5.4 Optimization Methods

Coliny method independent controls  The method independent controls for max_iterations and max_function_evaluations limit the number of major iterations and the number of function evaluations that can be performed during a Coliny optimization, respectively. The convergence_tolerance control defines the threshold value on relative change in the objective function that indicates convergence. The output verbosity specification controls the amount of information generated by Coliny: the silent, quiet, and normal settings correspond to minimal reporting from Coliny, whereas the verbose setting corresponds to a higher level of information, and debug outputs method initialization and a variety of internal Coliny diagnostics. The majority of Coliny’s methods perform independent function evaluations that can directly take advantage of DAKOTA’s parallel capabilities. Only coliny_solis_wets, coliny_cobyla, and certain configurations of coliny_pattern_search are inherently serial (see Pattern Search). The parallel methods automatically utilize parallel logic when the DAKOTA configuration supports parallelism. Lastly, neither speculative gradients nor linear constraints are currently supported with Coliny. Specification detail for method independent controls is provided in Tables 5.1 through 5.2.

Some COLINY methods exploit parallelism through the use of DAKOTA’s concurrent function evaluations. The nature of the algorithms, however, limits the amount of concurrency that can be exploited. The maximum amount of evaluation concurrency that can be leveraged by the various methods is as follows:

- COBYLA: one
- DIRECT: twice the number of variables
- Evolutionary Algorithms: size of the population
- Pattern Search: size of the search pattern
- Solis-Wets: one

Coliny method dependent controls  All Coliny methods support the show_misc_options optional specification which results in a dump of all the allowable method inputs. Note that the information provided by this command refers to optimizer parameters that are internal to Coliny, and which may differ from corresponding parameters used by the DAKOTA interface. The misc_options optional specification provides a means for inputing additional settings supported by the Coliny methods but which are not currently mapped through the DAKOTA input specification. Care must be taken in using this specification; they should only be employed by users familiar with the full range of parameter specifications available directly from Coliny and understand any differences that exist between those specifications and the ones available through DAKOTA.

Each of the Coliny methods supports the solution_target control, which defines a convergence criterion in which the optimizer will terminate if it finds an objective function value lower than the specified target. Specification detail for method dependent controls for all Coliny methods is provided in Table 5.9.

Each Coliny method supplements the settings of Table 5.9 with controls which are specific to its particular class of method.

COBYLA  The Constrained Optimization BY Linear Approximations (COBYLA) algorithm is an extension to the Nelder-Mead simplex algorithm for handling general linear/nonlinear constraints and is invoked using the coliny_cobyla group specification. The COBYLA algorithm employs linear approximations to the objective and constraint functions, the approximations being formed by linear interpolation at N+1 points in the space of the variables. We regard these interpolation points as vertices of a simplex. The step length parameter controls the size of the simplex and it is reduced automatically from initial_delta to threshold_delta. One advantage
that COBYLA has over many of its competitors is that it treats each constraint individually when calculating a change to the variables, instead of lumping the constraints together into a single penalty function.

COBYLA currently only supports termination based on the max_function_evaluations and solution_target specifications. The search performed by COBYLA is currently not parallelized.

Table 5.10 summarizes the COBYLA specification.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
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<td>Initial offset value</td>
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<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Threshold for offset values</td>
<td>threshold_-delta</td>
<td>real</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 5.10: Specification detail for the COBYLA method

**DIRECT** The DIviding RECTangles (DIRECT) optimization algorithm is a derivative free global optimization method that balances local search in promising regions of the design space with global search in unexplored regions. As shown in Figure 5.1, DIRECT adaptively subdivides the space of feasible design points so as to guarantee that iterates are generated in the neighborhood of a global minimum in finitely many iterations.
5.4 Optimization Methods

In practice, DIRECT has proven an effective heuristic for engineering design applications, for which it is able to quickly identify candidate solutions that can be further refined with fast local optimizers.

DIRECT uses the solution_target, constraint_penalty and show_misc_options specifications that are described in Coliny method dependent controls. Note, however, that DIRECT uses a fixed penalty value for constraint violations (i.e. it is not dynamically adapted as is done in coliny_pattern_search).

The division specification determines how DIRECT subdivides each subregion of the search space. If division is set to major_dimension, then the dimension representing the longest edge of the subregion is subdivided (this is the default). If division is set to all_dimensions, then all dimensions are simultaneously subdivided.

Each subregion considered by DIRECT has a size, which corresponds to the longest diagonal of the subregion. The global_balance_parameter controls how much global search is performed by only allowing a subregion to be subdivided if the size of the subregion divided by the size of the largest subregion is at least global_balance_parameter. Intuitively, this forces large subregions to be subdivided before the smallest subregions are refined. The local_balance_parameter provides a tolerance for estimating whether the smallest subregion can provide a sufficient decrease to be worth subdividing; the default value is a small value that is suitable for most applications.

DIRECT can be terminated with the standard max_function_evaluations and solution_target specifications. Additionally, the max_boxsize_limit specification terminates DIRECT if the size of the largest subregion falls below this threshold, and the min_boxsize_limit specification terminates DIRECT if the size of the smallest subregion falls below this threshold. In practice, this latter specification is likely to be more effective at limiting DIRECT’s search.

Table 5.11 summarizes the DIRECT specification.
Table 5.11: Specification detail for the DIRECT method

<table>
<thead>
<tr>
<th>Description</th>
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</thead>
<tbody>
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</tr>
<tr>
<td>Box subdivision approach</td>
<td>division</td>
<td>major_dimension</td>
<td>all_dimensions</td>
<td>Optional group</td>
</tr>
<tr>
<td>Global search balancing parameter</td>
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<td>Optional</td>
<td>0.0</td>
</tr>
<tr>
<td>Local search balancing parameter</td>
<td>local_balance_parameter</td>
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<td>Optional</td>
<td>1.e-8</td>
</tr>
<tr>
<td>Maximum boxsize limit</td>
<td>max_boxsize_limit</td>
<td>real</td>
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<tr>
<td>Minimum boxsize limit</td>
<td>min_boxsize_limit</td>
<td>real</td>
<td>Optional</td>
<td>0.0001</td>
</tr>
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<td>Constraint penalty</td>
<td>constraint_penalty</td>
<td>real</td>
<td>Optional</td>
<td>1000.0</td>
</tr>
</tbody>
</table>

Evolutionary Algorithms  DAKOTA currently provides several variants of evolutionary algorithms, invoked through the coliny_ea group specification.

The basic steps of an evolutionary algorithm are depicted in Figure 5.2.

Figure 5.2: Depiction of evolutionary algorithm
5.4 Optimization Methods

They can be enumerated as follows:

1. Select an initial population randomly and perform function evaluations on these individuals
2. Perform selection for parents based on relative fitness
3. Apply crossover and mutation to generate new_solutions_generated new individuals from the selected parents
   - Apply crossover with a fixed probability from two selected parents
   - If crossover is applied, apply mutation to the newly generated individual with a fixed probability
   - If crossover is not applied, apply mutation with a fixed probability to a single selected parent
4. Perform function evaluations on the new individuals
5. Perform replacement to determine the new population
6. Return to step 2 and continue the algorithm until convergence criteria are satisfied or iteration limits are exceeded

Table 5.12 provides the specification detail for the controls for seeding the method, initializing a population, and for selecting and replacing population members.

The random seed control provides a mechanism for making a stochastic optimization repeatable. That is, the use of the same random seed in identical studies will generate identical results. The population_size control specifies how many individuals will comprise the EA's population.

The initialization_type defines the type of initialization for the population of the EA. There are three types: simple_random, unique_random, and flat_file. simple_random creates initial solutions with random variable values according to a uniform random number distribution. It gives no consideration to any previously generated designs. The number of designs is specified by the population_size. unique_random is the same as simple_random, except that when a new solution is generated, it is checked against the rest of the solutions. If it duplicates any of them, it is rejected. flat_file allows the initial population to be read from a flat file. If flat_file is specified, a file name must be given.

The fitness_type controls how strongly differences in "fitness" (i.e., the objective function) are weighted in the process of selecting "parents" for crossover:

- the linear_rank setting uses a linear scaling of probability of selection based on the rank order of each individual’s objective function within the population
- the merit_function setting uses a proportional scaling of probability of selection based on the relative value of each individual’s objective function within the population

The replacement_type controls how current populations and newly generated individuals are combined to create a new population. Each of the replacement_type selections accepts an integer value, which is referred to below and in Table 5.12 as the replacement_size:

- The random setting creates a new population using (a) replacement_size randomly selected individuals from the current population, and (b) population_size - replacement_size individuals randomly selected from among the newly generated individuals (the number of which is optionally specified using new_solutions_generated) that are created for each generation (using the selection, crossover, and mutation procedures).
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
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<td>EA selection</td>
<td>coliny_ea</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Random seed</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>randomly generated seed</td>
</tr>
<tr>
<td>Number of population members</td>
<td>population_size</td>
<td>integer</td>
<td>Optional</td>
<td>50</td>
</tr>
<tr>
<td>Initialization type</td>
<td>initialization_type</td>
<td>simple_random</td>
<td>Required</td>
<td>unique_random</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unique_random</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>flat_file</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fitness type</td>
<td>fitness_type</td>
<td>linear_rank</td>
<td>Optional</td>
<td>linear_rank</td>
</tr>
<tr>
<td></td>
<td></td>
<td>merit_function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Replacement type</td>
<td>replacement_type</td>
<td>random</td>
<td>Optional group</td>
<td>elitist = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>chc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random replacement type</td>
<td>random</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>CHC replacement type</td>
<td>chc</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Elitist replacement type</td>
<td>elitist</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>New solutions generated</td>
<td>new_solutions_generated</td>
<td>integer</td>
<td>Optional</td>
<td>population_size - replacement_size</td>
</tr>
</tbody>
</table>

Table 5.12: Specification detail for the Coliny EA method dependent controls: seed, initialization, selection, and replacement
5.4 Optimization Methods

- The **chc** setting creates a new population using (a) the `replacement_size` best individuals from the combination of the current population and the newly generated individuals, and (b) `population_size - replacement_size` individuals randomly selected from among the remaining individuals in this combined pool. The chc setting is the preferred selection for many engineering problems.

- The **elitist** (default) setting creates a new population using (a) the `replacement_size` best individuals from the current population, (b) and `population_size - replacement_size` individuals randomly selected from the newly generated individuals. It is possible in this case to lose a good solution from the newly generated individuals if it is not randomly selected for replacement; however, the default `new_solutions_generated` value is set such that the entire set of newly generated individuals will be selected for replacement.

Note that `new_solutions_generated` is not recognized by DAKOTA as a valid keyword unless `replacement_type` has been specified.

Table 5.13 show the controls in the EA method associated with crossover and mutation.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
</table>
| Crossover type            | crossover_-
                          | type                         | Optional   | two_point           |
|                           |               | two_point| blend| uniform              |           |
| Crossover rate            | crossover_-
                          | rate                         | Optional   | 0.8                  |
|                           |               | real                         |            |                      |
| Mutation type             | mutation_-
                          | type                         | Optional   | offset_-
                          | normal |           |
|                           |               | replace_-
                          |           | uniform | offset_-
                          | normal |           |
|                           |               | offset_-
                          |           | cauchy  | offset_-
                          | uniform |           |
|                           |               | real                         | Optional   | 0.1                  |
| Mutation scale            | mutation_-
                          | scale                        | Optional   |                      |
|                           |               | real                         |            | 0.1                  |
| Mutation range            | mutation_-
                          | range                        | Optional   | 1                    |
|                           |               | integer                      |            |                      |
| Mutation dimension ratio  | dimension_-
                          | ratio                        | Optional   | 1.0                  |
|                           |               | real                         |            |                      |
| Mutation rate             | mutation_-
                          | rate                        | Optional   | 1.0                  |
|                           |               | real                         |            |                      |
| Non-adaptive mutation flag| non_adaptive  | none                         | Optional   | Adaptive mutation    |

Table 5.13: Specification detail for the Coliny EA method: crossover and mutation

The `crossover_type` controls what approach is employed for combining parent genetic information to create offspring, and the `crossover_rate` specifies the probability of a crossover operation being performed to generate a new offspring. The Coliny EA method supports three forms of crossover, two_point, blend, and uniform, which generate a new individual through combinations of two parent individuals. Two-point crossover divides each parent into three regions, where offspring are created from the combination of the middle region from one parent and the end regions from the other parent. Since the Coliny EA does not utilize bit
representations of variable values, the crossover points only occur on coordinate boundaries, never within the bits of a particular coordinate. Uniform crossover creates offspring through random combination of coordinates from the two parents. Blend crossover generates a new individual randomly along the multidimensional vector connecting the two parents.

The mutation_type controls what approach is employed in randomly modifying continuous design variables within the EA population. Each of the mutation methods generates coordinate-wise changes to individuals, usually by adding a random variable to a given coordinate value (an "offset" mutation), but also by replacing a given coordinate value with a random variable (a "replace" mutation). Discrete design variables are always mutated using the offset_uniform method. The mutation_rate controls the probability of mutation being performed on an individual, both for new individuals generated by crossover (if crossover occurs) and for individuals from the existing population. When mutation is performed, all dimensions of each individual are mutated. The mutation_scale specifies a scale factor which scales continuous mutation offsets; this is a fraction of the total range of each dimension, so mutation_scale is a relative value between 0 and 1. The mutation_range is used to control offset_uniform mutation used for discrete parameters. The replacement discrete value is the original value plus or minus an integer value up to mutation_range. The offset_normal, offset_cauchy, and offset_uniform mutation types are "offset" mutations in that they add a 0-mean random variable with a normal, cauchy, or uniform distribution, respectively, to the existing coordinate value. These offsets are limited in magnitude by mutation_scale. The replace_uniform mutation type is not limited by mutation_scale; rather it generates a replacement value for a coordinate using a uniformly distributed value over the total range for that coordinate.

Note that mutation_scale and mutation_range are not recognized by DAKOTA as valid keywords unless mutation_type has been specified and the type is an "offset" mutations.

The Coliny EA method uses self-adaptive mutation, which modifies the mutation scale dynamically. This mechanism is borrowed from EAs like evolution strategies. The non_adaptive flag can be used to deactivate the self-adaptation, which may facilitate a more global search.

Note that non_adaptive is not recognized by DAKOTA as a valid keyword unless mutation_type has been specified.

**Pattern Search** Pattern search techniques are nongradient-based optimization methods which use a set of offsets from the current iterate to locate improved points in the design space. The Coliny pattern search technique is invoked using a coliny_pattern_search group specification, which includes a variety of specification components.

Traditional pattern search methods search with a fixed pattern of search directions to try to find improvements to the current iterate. The Coliny pattern search methods generalize this simple algorithmic strategy to enable control of how the search pattern is adapted, as well as how each search pattern is evaluated. The stochastic and synchronization specifications denote how the the trial points are evaluated. The stochastic specification indicates that the trial points are considered in a random order. For parallel pattern search, synchronization dictates whether the evaluations are scheduled using a blocking scheduler or a nonblocking scheduler (i.e., Model::synchronize() or Model::synchronize_nowait(), respectively). In the blocking case, all points in the pattern are evaluated (in parallel), and if the best of these trial points is an improving point, then it becomes the next iterate. These runs are reproducible, assuming use of the same seed in the stochastic case. In the nonblocking case, all points in the pattern may not be evaluated, since the first improving point found becomes the next iterate. Since the algorithm steps will be subject to parallel timing variabilities, these runs will not generally be repeatable. The synchronization specification has similar connotations for sequential pattern search. If blocking is specified, then each sequential iteration terminates after all trial points have been considered, and if nonblocking is specified, then each sequential iteration terminates.
after the first improving trial point is evaluated. In this release, both blocking and nonblocking specifications result in blocking behavior (except in the case where exploratory_moves below is set to adaptive_pattern). Nonblocking behavior will be re-enabled after some underlying technical issues have been resolved.

The particular form of the search pattern is controlled by the pattern_basis specification. If pattern_basis is coordinate basis, then the pattern search uses a plus and minus offset in each coordinate direction, for a total of $2n$ function evaluations in the pattern. This case is depicted in Figure 5.3 for three coordinate dimensions.

If pattern_basis is simplex, then pattern search uses a minimal positive basis simplex for the parameter space, for a total of $n+1$ function evaluations in the pattern. Note that the simplex pattern basis can be used for unbounded problems only. The total_pattern_size specification can be used to augment the basic coordinate and simplex patterns with additional function evaluations, and is particularly useful for parallel load balancing. For example, if some function evaluations in the pattern are dropped due to duplication or bound constraint interaction, then the total_pattern_size specification instructs the algorithm to generate new offsets to bring the total number of evaluations up to this consistent total.

The exploratory_moves specification controls how the search pattern is adapted. (The search pattern can be adapted after an improving trial point is found, or after all trial points in a search pattern have been found to be unimproving points.) The following exploratory_moves selections are supported by Coliny:
• The basic_pattern case is the simple pattern search approach, which uses the same pattern in each iteration.

• The multi_step case examines each trial step in the pattern in turn. If a successful step is found, the pattern search continues examining trial steps about this new point. In this manner, the effects of multiple successful steps are cumulative within a single iteration. This option does not support any parallelism and will result in a serial pattern search.

• The adaptive_pattern case invokes a pattern search technique that adaptively rescales the different search directions to maximize the number of redundant function evaluations. See [Hart et al., 2001] for details of this method. In preliminary experiments, this method had more robust performance than the standard basic_pattern case in serial tests. This option supports a limited degree of parallelism. After successful iterations (where the step length is not contracted), a parallel search will be performed. After unsuccessful iterations (where the step length is contracted), only a single evaluation is performed.

The initial_delta and threshold_delta specifications provide the initial offset size and the threshold size at which to terminate the algorithm. For any dimension that has both upper and lower bounds, this step length will be internally rescaled to provide search steps of length initial_delta * range. This rescaling does not occur for other dimensions, so search steps in those directions have length initial_delta.

In general, pattern search methods can expand and contract their step lengths. Coliny pattern search methods contract the step length by the value contraction_factor, and they expand the step length by the value (1/contraction_factor). The expand_after_success control specifies how many successful objective function improvements must occur with a specific step length prior to expansion of the step length, whereas the no_expansion flag instructs the algorithm to forgo pattern expansion altogether.

Finally, constraint infeasibility can be managed in a somewhat more sophisticated manner than the simple weighted penalty function. If the constant_penalty specification is used, then the simple weighted penalty scheme described above is used. Otherwise, the constraint penalty is adapted to the value constraint_penalty/L, where L is the the smallest step length used so far.

Table 5.14 and Table 5.15 provide the specification detail for the Coliny pattern search method and its method dependent controls.

Solis-Wets  DAKOTA’s implementation of Coliny also contains the Solis-Wets algorithm. The Solis-Wets method is a simple greedy local search heuristic for continuous parameter spaces. Solis-Wets generates trial points using a multivariate normal distribution, and unsuccessful trial points are reflected about the current point to find a descent direction. This algorithm is inherently serial and will not utilize any parallelism. Table 5.16 provides the specification detail for this method and its method dependent controls.

These specifications have the same meaning as corresponding specifications for coliny_pattern_search. In particular, coliny_solis_wets supports dynamic rescaling of the step length, and dynamic rescaling of the constraint penalty. The only new specification is contract_after_failure, which specifies the number of unsuccessful cycles which must occur with a specific delta prior to contraction of the delta.

5.4.8 NCSU Methods

North Carolina State University (NCSU) has an implementation of the DIRECT algorithm (DIviding RECTangles algorithm that is outlined in the Coliny method section above). This version is documented in [Gablonsky,
## 5.4 Optimization Methods

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coliny pattern search method</td>
<td>coliny_pattern</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Stochastic pattern search</td>
<td>stochastic</td>
<td>none</td>
<td>Optional group</td>
<td>N/A</td>
</tr>
<tr>
<td>Random seed for stochastic pattern search</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>randomly generated seed</td>
</tr>
<tr>
<td>Initial offset value</td>
<td>initial_delta</td>
<td>real</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Threshold for offset values</td>
<td>threshold_delta</td>
<td>real</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Constraint penalty</td>
<td>constraint_penalty</td>
<td>real</td>
<td>Optional</td>
<td>1.0</td>
</tr>
<tr>
<td>Control of dynamic penalty</td>
<td>constant_penalty</td>
<td>none</td>
<td>Optional</td>
<td>algorithm dynamically adapts the constraint penalty</td>
</tr>
</tbody>
</table>

Table 5.14: Specification detail for the Coliny pattern search method: randomization, delta, and constraint controls

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Pattern basis selection</td>
<td>pattern_basis</td>
<td>coordinate</td>
<td>Optional</td>
<td>coordinate</td>
</tr>
<tr>
<td>Total number of points in pattern</td>
<td>total_pattern_size</td>
<td>integer</td>
<td>Optional</td>
<td>no augmentation of basic pattern</td>
</tr>
<tr>
<td>No expansion flag</td>
<td>no_expansion</td>
<td>none</td>
<td>Optional</td>
<td>algorithm may expand pattern size</td>
</tr>
<tr>
<td>Number of consecutive improvements before expansion</td>
<td>expand_after_success</td>
<td>integer</td>
<td>Optional</td>
<td>5</td>
</tr>
<tr>
<td>Pattern contraction factor</td>
<td>contraction_factor</td>
<td>real</td>
<td>Optional</td>
<td>0.5</td>
</tr>
<tr>
<td>Evaluation synchronization</td>
<td>synchronization</td>
<td>blocking</td>
<td>Optional</td>
<td>nonblocking</td>
</tr>
<tr>
<td>Exploratory moves selection</td>
<td>exploratory_moves</td>
<td>basic_pattern</td>
<td>Optional</td>
<td>basic_pattern</td>
</tr>
</tbody>
</table>

Table 5.15: Specification detail for the Coliny pattern search method: pattern controls
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
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<td>Coliny Solis-Wets method</td>
<td>coliny_-solis_wets</td>
<td>none</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Random seed for stochastic pattern search</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>randomly generated seed</td>
</tr>
<tr>
<td>Initial offset value</td>
<td>initial_-delta</td>
<td>real</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Threshold for offset values</td>
<td>threshold_-delta</td>
<td>real</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>No expansion flag</td>
<td>no_expansion</td>
<td>none</td>
<td>Optional</td>
<td>algorithm may expand pattern size</td>
</tr>
<tr>
<td>Number of consecutive improvements before expansion</td>
<td>expand_-after_-success</td>
<td>integer</td>
<td>Optional</td>
<td>5</td>
</tr>
<tr>
<td>Number of consecutive failures before contraction</td>
<td>contract_-after_-failure</td>
<td>integer</td>
<td>Optional</td>
<td>4*number of variables</td>
</tr>
<tr>
<td>Pattern contraction factor</td>
<td>contraction_-factor</td>
<td>real</td>
<td>Optional</td>
<td>0.5</td>
</tr>
<tr>
<td>Constraint penalty</td>
<td>constraint_-penalty</td>
<td>real</td>
<td>Optional</td>
<td>1.0</td>
</tr>
<tr>
<td>Control of dynamic penalty</td>
<td>constant_-penalty</td>
<td>none</td>
<td>Optional</td>
<td>algorithm dynamically adapts the constraint penalty</td>
</tr>
</tbody>
</table>

Table 5.16: Specification detail for the Coliny Solis-Wets method
5.4 Optimization Methods

We have found that the NCSU DIRECT implementation works better and is more robust for some problems than coliny_direct. Currently, we maintain both versions of DIRECT in DAKOTA; in the future, we may deprecate one. The NCSU DIRECT method is selected with ncsu_direct. We have tried to maintain consistency between the keywords in COLINY and NCSU implementation of DIRECT, but the algorithms have different parameters, so the keywords sometimes have slightly different meaning.

**NCSU method independent controls**  The method independent controls for max_iterations and max_function_evaluations limit the number of iterations and the number of function evaluations that can be performed during an NCSU DIRECT optimization. This methods will always strictly respect the number of iterations, but may slightly exceed the number of function evaluations, as it will always explore all sub-rectangles at the current level.

**NCSU method dependent controls**  There are four specification controls affecting NCSU DIRECT: solution_target, convergence_tolerance, min_boxsize_limit, and volume_boxsize_limit. The solution target specifies a goal toward which the optimizer should track. When solution_target is specified, convergence_tolerance specifies a percent error on the optimization. This is used for test problems, when the true global minimum is known (call it solution_target := fglobal). Then, the optimization terminates when 100(f_min-fglobal)/max(1,abs(fglobal) < convergence_tolerance. The default for fglobal is -1.0e100 and the default for convergence tolerance is as given above.

min_boxsize_limit is a setting that terminates the optimization when the measure of a hyperrectangle S with f(c(S)) = fmin is less than min_boxsize_limit. volume_boxsize_limit is a setting that terminates the optimization when the volume of a hyperrectangle S with f(c(S)) = fmin is less than volume_boxsize_limit percent of the original hyperrectangle. Basically, volume_boxsize_limit stops the optimization when the volume of the particular rectangle which has fmin is less than a certain percentage of the whole volume. min_boxsize_limit uses an arbitrary measure to stop the optimization. The keywords for NCSU DIRECT are described in Table 5.17 below.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution Target</td>
<td>solution_target</td>
<td>real</td>
<td>Optional</td>
<td>0</td>
</tr>
<tr>
<td>Min boxsize limit</td>
<td>min_boxsize_limit</td>
<td>real in [0,1]</td>
<td>Optional</td>
<td>1.0e-4</td>
</tr>
<tr>
<td>Volume boxsize limit</td>
<td>vol_boxsize_limit</td>
<td>real in [0,1]</td>
<td>Optional</td>
<td>1.0e-6</td>
</tr>
</tbody>
</table>

Table 5.17: Specification detail for the NCSU DIRECT method

5.4.9 JEGA Methods

The JEGA library [Eddy and Lewis, 2001] contains two global optimization methods. The first is a Multi-objective Genetic Algorithm (MOGA) which performs Pareto optimization. The second is a Single-objective Genetic Algorithm (SOGA) which performs optimization on a single objective function. Both methods support general constraints and a mixture of real and discrete variables. The JEGA library was written by John Eddy, currently a member of the technical staff in the System Readiness and Sustainment Technologies department at Sandia.
National Laboratories in Albuquerque. These algorithms are accessed as moga and soga within DAKOTA. DAKOTA provides access to the JEGA library through the JEGAOptimizer class.

**JEGA method independent controls**  JEGA utilizes the max_iterations and max_function_evaluations method independent controls to provide integer limits for the maximum number of generations and function evaluations, respectively. Note that currently, the DAKOTA default for max_iterations is 100 and for max_function_evaluations is 1000. These are the default settings that will be used to "stop" the JEGA algorithms, unless some specific convergence criteria are set (see Tables 5.20 and 5.21 below).

Beginning with v2.0, JEGA also utilizes the output method independent control to vary the amount of information presented to the user during execution.

**JEGA method dependent controls**  The JEGA library currently provides two types of genetic algorithms (GAs): a multi-objective genetic algorithm (moga), and a single-objective genetic algorithm (soga). Both of these GAs can take real-valued inputs, integer-valued inputs, or a mixture of real and integer-valued inputs. "Real-valued" and "integer-valued" refer to the use of continuous or discrete variable domains, respectively (the response data are real-valued in all cases).

The basic steps of the genetic algorithm are as follows:

1. Initialize the population (by randomly generating population members with or without duplicates allowed, or by flat-file initialization)
2. Evaluate the initial population members (calculate the values of the objective function(s) and constraints for each population member)
3. Perform crossover (several crossover types are available)
4. Perform mutation (several mutation types are available)
5. Evaluate the new population members.
6. Assess the fitness of each member in the population. There are a number of ways to evaluate the fitness of the members of the population. Choice of fitness assessor operators is strongly related to the type of replacement algorithm being used and can have a profound effect on the solutions selected for the next generation. For example, if using MOGA, the available assessors are the layer_rank and domination_count fitness assessors. If using either of these, it is strongly recommended that you use the replacement_type called the below_limit selector as well (although the roulette wheel selectors can also be used). The functionality of the domination_count selector of JEGA v1.0 can now be achieved using the domination_count fitness assessor and below_limit replacement selector together. If using SOGA, there are a number of possible combinations of fitness assessors and selectors.
7. Replace the population with members selected to continue in the next generation. The pool of potential members is the current population and the current set of offspring. The replacement_type of roulette_wheel or unique_roulette_wheel may be used either with MOGA or SOGA problems however they are not recommended for use with MOGA. Given that the only two fitness assessors for MOGA are the layer_rank and domination_count, the recommended selector is the below_limit selector. The below_limit replacement will only keep designs that are dominated by fewer than a limiting number of other designs. The replacement_type of favor_feasible is specific to a SOGA. This replacement operator will always prefer a more feasible design to a less feasible one. Beyond that, it favors solutions based on an assigned fitness value which must have been installed by the weighted sum only fitness assessor (see the discussion below).
8. Apply niche pressure to the population. This step is specific to the MOGA and is new as of JEGA v2.0. Technically, the step is carried out during runs of the SOGA but only the null_niching operator is available for use with SOGA. In MOGA, the radial or distance operators can be used. The purpose of niching is to encourage differentiation along the Pareto frontier and thus a more even and uniform sampling. The radial nicher takes information input from the user to compute a minimum allowable distance between designs in the performance space and acts as a secondary selection operator whereby it enforces this minimum distance. The distance nicher requires that solutions must be separated from other solutions by a minimum distance in each dimension (vs. Euclidean distance for the radial niching). After niching is complete, all designs in the population will be at least the minimum distance from one another in all directions.

9. Test for convergence. There are two aspects to convergence that must be considered. The first is stopping criteria. A stopping criteria dictates some sort of limit on the algorithm that is independent of its performance. Examples of stopping criteria available for use with JEGA are the max_iterations and max_function_evaluations inputs. All JEGA convergers respect these stopping criteria in addition to anything else that they do. The second aspect to convergence involves repeated assessment of the algorithms progress in solving the problem. In JEGA v1.0, the SOGA fitness tracker convergers (best_fitness_tracker and average_fitness_tracker) performed this function by asserting that the fitness values (either best or average) of the population continue to improve. There was no such operator for the MOGA. As of JEGA v2.0, the same fitness tracker convergers exist for use with SOGA and there is now a converger available for use with the MOGA. The MOGA converger (metric_tracker) operates by tracking various changes in the non-dominated frontier from generation to generation. When the changes occurring over a user specified number of generations fall below a user specified threshold, the algorithm stops.

10. Perform post processing. This step is new as of JEGA v2.1. The purpose of this operation is to perform any needed data manipulations on the final solution deemed necessary. Currently the distance_postprocessor is the only one other than the null_postprocessor. The distance_postprocessor is specifically for use with the MOGA and reduces the final solution set size such that a minimum distance in each direction exists between any two designs.

There are many controls which can be used for both MOGA and SOGA methods. These include among others the random seed, initialization types, crossover and mutation types, and some replacement types. These are described in Tables 5.18 and 5.19 below.

The seed control defines the starting seed for the random number generator. The algorithm uses random numbers heavily but a specification of a random seed will cause the algorithm to run identically from one trial to the next so long as all other input specifications remain the same. New as of JEGA v2.0 is the introduction of the log_file specification. JEGA now uses a logging library to output messages and status to the user. JEGA can be configured at build time to log to both the console window and a text file, one or the other, or neither. The log_file input is a string name of a file into which to log. If the build was configured without file logging in JEGA, this input is ignored. If file logging is enabled and no log_file is specified, the default file name of JEGAGlobal.log is used. Also new to JEGA v2.0 is the introduction of the print_each_pop specification. It serves as a flag and if supplied, the population at each generation will be printed to a file named "population<GEN#>.dat" where <GEN#> is the number of the current generation.

The initialization_type defines the type of initialization for the GA. There are three types: simple_random, unique_random, and flat_file. simple_random creates initial solutions with random variable values according to a uniform random number distribution. It gives no consideration to any previously generated designs. The number of designs is specified by the population_size. unique_random is the same as simple_random, except that when a new solution is generated, it is checked against the rest of the
solutions. If it duplicates any of them, it is rejected. flat_file allows the initial population to be read from
a flat file. If flat_file is specified, a file name must be given. Variables can be delimited in the flat file in
any way you see fit with a few exceptions. The delimiter must be the same on any given line of input with the
exception of leading and trailing whitespace. So a line could look like: 1.1, 2.2, 3.3 for example but could not
look like: 1.1, 2.2 3.3. The delimiter can vary from line to line within the file which can be useful if data from
multiple sources is pasted into the same input file. The delimiter can be any string that does not contain any of
the characters .+dDeE or any of the digits 0-9. The input will be read until the end of the file. The algorithm will
discard any configurations for which it was unable to retrieve at least the number of design variables. The objective
and constraint entries are not required but if ALL are present, they will be recorded and the design will be
tagged as evaluated so that evaluators may choose not to re-evaluate them. Setting the size for this initializer has
the effect of requiring a minimum number of designs to create. If this minimum number has not been created once
the files are all read, the rest are created using the unique_random initializer and then the simple_random
initializer if necessary.

Note that the population_size only sets the size of the initial population. The population size may vary in
the JEGA methods according to the type of operators chosen for a particular optimization run.

There are many crossover types available. multi_point_binary crossover requires an integer number, N, of
crossover points. This crossover type performs a bit switching crossover at N crossover points in the binary en-
coded genome of two designs. Thus, crossover may occur at any point along a solution chromosome (in the middle
of a gene representing a design variable, for example). multi_point_parameterized_binary crossover is simi-
lar in that it performs a bit switching crossover routine at N crossover points. However, this crossover type
performs crossover on each design variable individually. So the individual chromosomes are crossed at N loca-
tions. multi_point_real crossover performs a variable switching crossover routing at N crossover points
in the real real valued genome of two designs. In this scheme, crossover only occurs between design variables
(chromosomes). Note that the standard solution chromosome representation in the JEGA algorithm is real en-
coded and can handle integer or real design variables. For any crossover types that use a binary representation,
real variables are converted to long integers by multiplying the real number by 10^{6} and then truncating. Note
that this assumes a precision of only six decimal places. Discrete variables are represented as integers (indices
within a list of possible values) within the algorithm and thus require no special treatment by the binary operators.

The final crossover type is shuffle_random. This crossover type performs crossover by choosing design
variables at random from a specified number of parents enough times that the requested number of children are
produced. For example, consider the case of 3 parents producing 2 children. This operator would go through and
for each design variable, select one of the parents as the donor for the child. So it creates a random shuffle of the
parent design variable values. The relative numbers of children and parents are controllable to allow for as much
mixing as desired. The more parents involved, the less likely that the children will wind up exact duplicates of the
parents.

All crossover types take a crossover_rate. The crossover rate is used to calculate the number of crossover
operations that take place. The number of crossovers is equal to the rate * population_size.

There are five mutation types allowed. replace_uniform introduces random variation by first randomly
choosing a design variable of a randomly selected design and reassigning it to a random valid value for that
variable. No consideration of the current value is given when determining the new value. All mutation types have
a mutation_rate. The number of mutations for the replace_uniform mutator is the product of the mutation_rate
and the population_size.

The bit_random mutator introduces random variation by first converting a randomly chosen variable of a
randomly chosen design into a binary string. It then flips a randomly chosen bit in the string from a 1 to a 0 or
visa versa. In this mutation scheme, the resulting value has more probability of being similar to the original value.
The number of mutations performed is the product of the mutation_rate, the number of design variables, and the
population size.
The offset mutators all act by adding an "offset" random amount to a variable value. The random amount has a mean of zero in all cases. The offset_normal mutator introduces random variation by adding a Gaussian random amount to a variable value. The random amount has a standard deviation dependent on the mutation_scale. The mutation_scale is a fraction in the range [0, 1] and is meant to help control the amount of variation that takes place when a variable is mutated. mutation_scale is multiplied by the range of the variable being mutated to serve as standard deviation. offset_cauchy is similar to offset_normal, except that a Cauchy random variable is added to the variable being mutated. The mutation_scale also defines the standard deviation for this mutator. Finally, offset_uniform adds a uniform random amount to the variable value. For the offset_uniform mutator, the mutation_scale is interpreted as a fraction of the total range of the variable. The range of possible deviation amounts is +/- 1/2 *(mutation_scale * variable range). The number of mutations for all offset mutators is defined as the product of mutation_rate and population_size.

As of JEGA v2.0, all replacement types are common to both MOGA and SOGA. They include the roulette_wheel, unique_roulette_wheel, elitist, and below_limit selectors. In roulette_wheel replacement, each design is conceptually allotted a portion of a wheel proportional to its fitness relative to the fitnesses of the other Designs. Then, portions of the wheel are chosen at random and the design occupying those portions are duplicated into the next population. Those Designs allotted larger portions of the wheel are more likely to be selected (potentially many times). unique_roulette_wheel replacement is the same as roulette_wheel replacement, with the exception that a design may only be selected once. The below_limit selector attempts to keep all designs for which the negated fitness is below a certain limit. The values are negated to keep with the convention that higher fitness is better. The inputs to the below_limit selector are the limit as a real value, and a shrinkage_percentage as a real value. The shrinkage_percentage defines the minimum amount of selections that will take place if enough designs are available. It is interpreted as a percentage of the population size that must go on to the subsequent generation. To enforce this, below_limit makes all the selections it would make anyway and if that is not enough, it takes the remaining that it needs from the best of what is left (effectively raising its limit as far as it must to get the minimum number of selections). It continues until it has made enough selections. The shrinkage_percentage is designed to prevent extreme decreases in the population size at any given generation, and thus prevent a big loss of genetic diversity in a very short time. Without a shrinkage limit, a small group of "super" designs may appear and quickly cull the population down to a size on the order of the limiting value. In this case, all the diversity of the population is lost and it is expensive to re-diversify and spread the population. The elitist selector simply chooses the required number of designs taking the most fit. For example, if 100 selections are requested, then the top 100 designs as ranked by fitness will be selected and the remaining will be discarded.

Multi-objective Evolutionary Algorithms  The specification for controls specific to Multi-objective Evolutionary algorithms are described here. These controls will be appropriate to use if the user has specified moga as the method.

The initialization, crossover, and mutation controls were all described in the preceding section. There are no MOGA specific aspects to these controls. The fitness_type for a MOGA may be domination_count or layer_rank. Both have been specifically designed to avoid problems with aggregating and scaling objective function values and transforming them into a single objective. Instead, the domination_count fitness assessor works by ordering population members by the negative of the number of designs that dominate them. The values are negated in keeping with the convention that higher fitness is better. The layer_rank fitness assessor works by assigning all non-dominated designs a layer of 0, then from what remains, assigning all the non-dominated a layer of 1, and so on until all designs have been assigned a layer. Again, the values are negated for the higher-is-better fitness convention. Use of the below_limit selector with the domination_count fitness assessor has the effect of keeping all designs that are dominated by fewer then a limiting number of other designs subject to the shrinkage limit. Using it with the layer_rank fitness assessor has the effect of keeping all those designs...
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
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<td>moga</td>
<td>soga</td>
<td>none</td>
<td>Required group</td>
</tr>
<tr>
<td>Random seed</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>Time based seed</td>
</tr>
<tr>
<td>Log file</td>
<td>log_file</td>
<td>string</td>
<td>Optional</td>
<td>JEGAGlobal.log</td>
</tr>
<tr>
<td>Number of population members</td>
<td>population_size</td>
<td>integer</td>
<td>Optional</td>
<td>50</td>
</tr>
<tr>
<td>Population output</td>
<td>print_each_pop</td>
<td>none</td>
<td>Optional</td>
<td>No printing</td>
</tr>
<tr>
<td>Output verbosity</td>
<td>output</td>
<td>silent</td>
<td>quiet</td>
<td>verbose</td>
</tr>
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<td>Initialization type</td>
<td>initialization_type</td>
<td>simple_random</td>
<td>unique_random</td>
<td>flat_file</td>
</tr>
<tr>
<td>Mutation type</td>
<td>mutation_type</td>
<td>replace_uniform</td>
<td>bit_random</td>
<td>offset_cauchy</td>
</tr>
<tr>
<td>Mutation scale</td>
<td>mutation_scale</td>
<td>real</td>
<td>Optional</td>
<td>0.15</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>mutation_rate</td>
<td>real</td>
<td>Optional</td>
<td>0.08</td>
</tr>
<tr>
<td>Replacement type</td>
<td>replacement_type</td>
<td>below_limit</td>
<td>roulette_wheel</td>
<td>unique_roulette_wheel</td>
</tr>
<tr>
<td>Below limit selection</td>
<td>below_limit</td>
<td>real</td>
<td>Optional</td>
<td>6</td>
</tr>
<tr>
<td>Shrinkage percentage in below limit selection</td>
<td>shrinkage_per-</td>
<td>real</td>
<td>Optional</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 5.18: Specification detail for JEGA method dependent controls: seed, output, initialization, mutation, and replacement
### Table 5.19: Specification detail for JEGA method dependent controls: crossover

<table>
<thead>
<tr>
<th>Description</th>
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<td>multi-point-parameterized_binary</td>
<td>multi-point_real</td>
</tr>
<tr>
<td>Multi point binary crossover</td>
<td>multi-point_binary</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Multi point parameterized binary crossover</td>
<td>multi-point-parameterized-binary</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Multi point real crossover</td>
<td>multi-point_real</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Random shuffle crossover</td>
<td>shuffle_random</td>
<td>num_parents, num_offspring</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of parents in random shuffle crossover</td>
<td>num_parents</td>
<td>integer</td>
<td>optional</td>
<td>2</td>
</tr>
<tr>
<td>Number of offspring in random shuffle crossover</td>
<td>num_offspring</td>
<td>integer</td>
<td>optional</td>
<td>2</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>crossover_rate</td>
<td>real</td>
<td>optional (applies to all crossover types)</td>
<td>0.8</td>
</tr>
</tbody>
</table>
whose layer is below a certain threshold again subject to the shrinkage limit.

New as of JEGA v2.0 is the introduction of niche pressure operators. These operators are meant primarily for use with the moga. The job of a niche pressure operator is to encourage diversity along the Pareto frontier as the algorithm runs. This is typically accomplished by discouraging clustering of design points in the performance space. In JEGA, the application of niche pressure occurs as a secondary selection operation. The nicher is given a chance to perform a pre-selection operation prior to the operation of the selection (replacement) operator, and is then called to perform niching on the set of designs that were selected by the selection operator.

Currently, the only niche pressure operators available are the radial nicher and the distance nicher. The radial niche pressure applicator works by enforcing a minimum Euclidean distance between designs in the performance space at each generation. The algorithm proceeds by starting at the (or one of the) extreme designs along objective dimension 0 and marching through the population removing all designs that are too close to the current design. One exception to the rule is that the algorithm will never remove an extreme design which is defined as a design that is maximal or minimal in all but 1 objective dimension (for a classical 2 objective problem, the extreme designs are those at the tips of the non-dominated frontier). The distance nicher enforces a minimum distance in each dimension.

The designs that are removed by the nicher are not discarded. They are buffered and re-inserted into the population during the next pre-selection operation. This way, the selector is still the only operator that discards designs and the algorithm will not waste time “re-filling” gaps created by the nicher.

The radial nicher requires as input a vector of fractions with length equal to the number of objectives. The elements of the vector are interpreted as percentages of the non-dominated range for each objective defining a minimum distance to all other designs. All values should be in the range (0, 1). The minimum allowable distance between any two designs in the performance space is the Euclidian (simple square-root-sum-of-squares calculation) distance defined by these percentages. The distance nicher has a similar input vector requirement, only the distance is the minimum distance in each dimension.

Also new as of JEGA v2.0 is the introduction of the MOGA specific metric_tracker converger. This converger is conceptually similar to the best and average fitness tracker convergers in that it tracks the progress of the population over a certain number of generations and stops when the progress falls below a certain threshold. The implementation is quite different however. The metric_tracker converger tracks 3 metrics specific to the non-dominated frontier from generation to generation. All 3 of these metrics are computed as percent changes between the generations. In order to compute these metrics, the converger stores a duplicate of the non-dominated frontier at each generation for comparison to the non-dominated frontier of the next generation.

The first metric is one that indicates how the expanse of the frontier is changing. The expanse along a given objective is defined by the range of values existing within the non-dominated set. The expansion metric is computed by tracking the extremes of the non-dominated frontier from one generation to the next. Any movement of the extreme values is noticed and the maximum percentage movement is computed as:

$$ Em = \max_{j} \abs{(\text{range}(j, i) - \text{range}(j, i-1)) / \text{range}(j, i-1)} $$

where Em is the max expansion metric, j is the objective function index, i is the current generation number, and nof is the total number of objectives. The range is the difference between the largest value along an objective and the smallest when considering only non-dominated designs.

The second metric monitors changes in the density of the non-dominated set. The density metric is computed as the number of non-dominated points divided by the hypervolume of the non-dominated region of space. Therefore, changes in the density can be caused by changes in the number of non-dominated points or by changes in size of the non-dominated space or both. The size of the non-dominated space is computed as:

$$ Vps(i) = \prod_{j} \text{range}(j, i) $$

where $Vps(i)$ is the hypervolume of the non-dominated set at generation $i$. The hypervolume is the product of the ranges along each objective.
where $V_{ps(i)}$ is the hypervolume of the non-dominated space at generation $i$ and all other terms have the same meanings as above.

The density of the given non-dominated space is then:

$$D_{ps(i)} = \frac{Pct(i)}{V_{ps(i)}}$$

where $Pct(i)$ is the number of points on the non-dominated frontier at generation $i$.

The percentage increase in density of the frontier is then calculated as

$$Cd = \frac{|D_{ps(i)} - D_{ps(i-1)}|}{D_{ps(i-1)}}$$

where $Cd$ is the change in density metric.

The final metric is one that monitors the "goodness" of the non-dominated frontier. This metric is computed by considering each design in the previous population and determining if it is dominated by any designs in the current population. All that are determined to be dominated are counted. The metric is the ratio of the number that are dominated to the total number that exist in the previous population.

As mentioned above, each of these metrics is a percentage. The tracker records the largest of these three at each generation. Once the recorded percentage is below the supplied percent change for the supplied number of generations consecutively, the algorithm is converged.

The specification for convergence in a moga can either be `metric_tracker` or can be omitted altogether. If omitted, no convergence algorithm will be used and the algorithm will rely on stopping criteria only. If `metric_tracker` is specified, then a `percent_change` and `num_generations` must be supplied as with the other metric tracker convergence (average and best fitness trackers). The `percent_change` is the threshold beneath which convergence is attained whereby it is compared to the metric value computed as described above. The `num_generations` is the number of generations over which the metric value should be tracked. Convergence will be attained if the recorded metric is below `percent_change` for `num_generations` consecutive generations.

The MOGA specific controls are described in Table 5.20 below. Note that MOGA and SOGA create additional output files during execution. "finaldata.dat" is a file that holds the final set of Pareto optimal solutions after any post-processing is complete. "discards.dat" holds solutions that were discarded from the population during the course of evolution. It can often be useful to plot objective function values from these files to visually see the Pareto front and ensure that finaldata.dat solutions dominate discards.dat solutions. The solutions are written to these output files in the format "Input1...InputN,Output1...OutputM". If MOGA is used in a hybrid optimization strategy (which requires one optimal solution from each individual optimization method to be passed to the subsequent optimization method as its starting point), the solution in the Pareto set closest to the "utopia" point is given as the best solution. This solution is also reported in the DAKOTA output. This "best" solution in the Pareto set has minimum distance from the utopia point. The utopia point is defined as the point of extreme (best) values for each objective function. For example, if the Pareto front is bounded by (1,100) and (90,2), then (1,2) is the utopia point. There will be a point in the Pareto set that has minimum L2-norm distance to this point, for example (10,10) may be such a point. In SOGA, the solution that minimizes the single objective function is returned as the best solution. If moga is used in a strategy which may require passing multiple solutions to the next level (such as the surrogate_based_global method or hybrid strategy), the orthogonal_distance postprocessor type may be used to specify the distances between each solution value to winnow down the solutions in the full Pareto front to a subset which will be passed to the next iteration.
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
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<td>Fitness type</td>
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</tr>
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<td>Niche pressure type</td>
<td>niching_type</td>
<td>radial</td>
<td>distance</td>
<td>Optional group</td>
</tr>
<tr>
<td>Niching distance</td>
<td>radial</td>
<td>distance</td>
<td>list of real</td>
<td>Optional</td>
</tr>
<tr>
<td>Convergence type</td>
<td>metric_tracker</td>
<td>none</td>
<td>Optional group</td>
<td>Stopping criteria only</td>
</tr>
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<td>real</td>
<td>Optional</td>
<td>0.1</td>
</tr>
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<td>Number generations for metric_tracker converger</td>
<td>numGenerations</td>
<td>integer</td>
<td>Optional</td>
<td>10</td>
</tr>
<tr>
<td>Post_processor type</td>
<td>postprocessor_type</td>
<td>orthogonal_distance</td>
<td>Optional</td>
<td>No post-processing of solutions</td>
</tr>
<tr>
<td>Post_processor distance</td>
<td>orthogonal_distance</td>
<td>list of real</td>
<td>Optional</td>
<td>0.01 for all objectives</td>
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</table>

Table 5.20: Specification detail for MOGA method controls
5.5 Least Squares Methods

**Single-objective Evolutionary Algorithms** The specification for controls specific to Single-objective Evolutionary algorithms are described here. These controls will be appropriate to use if the user has specified soga as the method.

The initialization, crossover, and mutation controls were all described above. There are no SOGA specific aspects to these controls. The replacement_type for a SOGA may be roulette_wheel, unique_roulette_wheel, elitist, or favor_feasible. The favor_feasible replacement type first considers feasibility as a selection criteria. If that does not produce a “winner” then it moves on to considering fitness value. Because of this, any fitness assessor used with the favor_feasible selector must only account objectives in the creation of fitness. Therefore, there is such a fitness assessor and it’s use is enforced when the \ favor_feasible selector is chosen. In that case, and if the output level is set high enough, a message will be presented indicating that the weighted_sum_only fitness assessor will be used. As of JECA v2.0 and beyond, the fitness assessment operator must be specified with SOGA although the merit_function is currently the only one (note that the weighted_sum_only assessor exists but cannot be selected). The roulette wheel selectors no longer assume a fitness function. The merit_function fitness assessor uses an exterior penalty function formulation to penalize infeasible designs. The specification allows the input of a constraint_penalty which is the multiplier to use on the constraint violations.

The SOGA controls allow two additional convergence types. The convergence_type called average_fitness_tracker keeps track of the average fitness in a population. If this average fitness does not change more than percent_change over some number of generations, num_generations, then the solution is reported as converged and the algorithm terminates. The best_fitness_tracker works in a similar manner, only it tracks the best fitness in the population. Convergence occurs after num_generations has passed and there has been less than percent_change in the best fitness value. The percent change can be as low as 0% in which case there must be no change at all over the number of generations. Both also respect the stopping criteria.

The SOGA specific controls are described in Table 5.21 below.

5.5 Least Squares Methods

DAKOTA’s least squares branch currently contains three methods for solving nonlinear least squares problems: NL2SOL, a trust-region method that adaptively chooses between two Hessian approximations (Gauss-Newton and Gauss-Newton plus a quasi-Newton approximation to the rest of the Hessian), NLSSOL, a sequential quadratic programming (SQP) approach that is from the same algorithm family as NPSOL, and Gauss-Newton, which supplies the Gauss-Newton Hessian approximation to the full-Newton optimizers from OPT++.

The important difference of these algorithms from general-purpose optimization methods is that the response set is defined by least squares terms, rather than an objective function. Thus, a finer granularity of data is used by least squares solvers as compared to that used by optimizers. This allows the exploitation of the special structure provided by a sum of squares objective function. Refer to Least squares terms and constraint functions (least squares data set) for additional information on the least squares response data set.

5.5.1 NL2SOL Method

NL2SOL is available as nl2sol and addresses unconstrained and bound-constrained problems. It uses a trust-region method (and thus can be viewed as a generalization of the Levenberg-Marquardt algorithm) and adaptively chooses between two Hessian approximations, the Gauss-Newton approximation alone and the Gauss-Newton approximation plus a quasi-Newton approximation to the rest of the Hessian. Even on small-residual problems, the latter Hessian approximation can be useful when the starting guess is far from the solution. On problems that
<table>
<thead>
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<td>fitness_type</td>
<td>merit_function</td>
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<td>merit_function</td>
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<td>Constraint penalty in merit function</td>
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</tr>
<tr>
<td>Replacement type</td>
<td>replacement_type</td>
<td>favor_feasible</td>
<td>unique_roulette_wheel</td>
<td>Required group</td>
</tr>
<tr>
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<td>best_fitness_tracker</td>
<td>average_fitness_tracker</td>
<td>Optional</td>
</tr>
<tr>
<td>Number of generations (for convergence test)</td>
<td>num_generations</td>
<td>integer</td>
<td>Optional</td>
<td>10</td>
</tr>
<tr>
<td>Percent change in fitness</td>
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<td>real</td>
<td>Optional</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5.21: Specification detail for SOGA method controls
are not over-parameterized (i.e., that do not involve more optimization variables than the data support), NL2SOL usually exhibits fast convergence.

NL2SOL has a variety of internal controls as described in AT&T Bell Labs CS TR 153 (http://cm.bell-labs.com/cm/cs/cstr/153.ps.gz). A number of existing DAKOTA controls (method independent controls and responses controls) are mapped into these NL2SOL internal controls. In particular, DAKOTA’s convergence_tolerance, max_iterations, max_function_evaluations, and fd_gradient_step_size are mapped directly into NL2SOL’s rfctol, mxiter, mxfcal, and dltfdj controls, respectively. In addition, DAKOTA’s fd_hessian_step_size is mapped into both delta0 and dltfdc, and DAKOTA’s output verbosity is mapped into NL2SOL’s auxprt and outlev (for normal/verbose/debug output, NL2SOL prints initial guess, final solution, solution statistics, nondefault values, and changes to the active bound constraint set on every iteration; for quiet output, NL2SOL prints only the initial guess and final solution; and for silent output, NL2SOL output is suppressed).

Several NL2SOL convergence tolerances are adjusted in response to function_precision, which gives the relative precision to which responses are computed. These tolerances may also be specified explicitly: convergence_tolerance (NL2SOL’s rfctol, as mentioned previously) is the relative-function convergence tolerance (on the accuracy desired in the sum-of-squares function); x_conv_tol (NL2SOL’s xctol) is the X-convergence tolerance (scaled relative accuracy of the solution variables); absolute_conv_tol (NL2SOL’s afctol) is the absolute function convergence tolerance (stop when half the sum of squares is less than absolute_conv_tol, which is mainly of interest on zero-residual test problems); singular_conv_tol (NL2SOL’s scctol) is the singular convergence tolerance, which works in conjunction with singular_radius (NL2SOL’s lmaxs) to test for underdetermined least-squares problems (stop when the relative reduction yet possible in the sum of squares appears less then singular_conv_tol for steps of scaled length at most singular_radius); false_conv_tol (NL2SOL’s xftol) is the false-convergence tolerance (stop with a suspicion of discontinuity when a more favorable stopping test is not satisfied and a step of scaled length at most false_conv_tol is not accepted). Finally, the initial_trust_radius specification (NL2SOL’s lmax0) specifies the initial trust region radius for the algorithm.

The internal NL2SOL defaults can be obtained for many of these controls by specifying the value -1. For both the singular_radius and the initial_trust_radius, this results in the internal use of steps of length 1. For other controls, the internal defaults are often functions of machine epsilon (as limited by function_precision). Refer to CS TR 153 for additional details on these formulations.

Whether and how NL2SOL computes and prints a final covariance matrix and regression diagnostics is affected by several keywords. covariance (NL2SOL’s covreq) specifies the desired covariance approximation:

- 0 = default = none
- 1 or -1 ==> \( \sigma^2 H^{-1} J^T J H^{-1} \)
- 2 or -2 ==> \( \sigma^2 H^{-1} \)
- 3 or -3 ==> \( \sigma^2 (J^T J)^{-1} \)

- Negative values ==> estimate the final Hessian H by finite differences of function values only (using fd_hessian_step_size)
- Positive values ==> differences of gradients (using fd_hessian_step_size)

When regression_diagnostics (NL2SOL’s rdreq) is specified and a positive-definite final Hessian approximation H is computed, NL2SOL computes and prints a regression diagnostic vector RD such that if omitting the i-th observation would cause alpha times the change in the solution that omitting the j-th observation would
cause, then $RD[i] = |\alpha| RD[j]$. The finite-difference step-size tolerance affecting $H$ is $fd\_hessian\_step\_size$ (NL2SOL’s $delta0$ and $dltfdc$, as mentioned previously).

Table 5.22 provides the specification detail for the NL2SOL method dependent controls.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative precision in least squares terms</td>
<td>function__precision</td>
<td>real</td>
<td>Optional</td>
<td>1e-10</td>
</tr>
<tr>
<td>Absolute function convergence tolerance</td>
<td>absolute__conv_tol</td>
<td>real</td>
<td>Optional</td>
<td>-1. (use NL2SOL internal default)</td>
</tr>
<tr>
<td>Convergence tolerance for change in parameter vector</td>
<td>x_conv_tol</td>
<td>real</td>
<td>Optional</td>
<td>-1. (use NL2SOL internal default)</td>
</tr>
<tr>
<td>Singular convergence tolerance</td>
<td>singular__conv_tol</td>
<td>real</td>
<td>Optional</td>
<td>-1. (use NL2SOL internal default)</td>
</tr>
<tr>
<td>Step limit for sctol</td>
<td>singular__radius</td>
<td>real</td>
<td>Optional</td>
<td>-1. (use NL2SOL internal default of 1)</td>
</tr>
<tr>
<td>False convergence tolerance</td>
<td>false_conv_tol</td>
<td>real</td>
<td>Optional</td>
<td>-1. (use NL2SOL internal default)</td>
</tr>
<tr>
<td>Initial trust region radius</td>
<td>initial__trust_radius</td>
<td>real</td>
<td>Optional</td>
<td>-1. (use NL2SOL internal default of 1)</td>
</tr>
<tr>
<td>Covariance post-processing</td>
<td>covariance</td>
<td>integer</td>
<td>Optional</td>
<td>0 (no covariance)</td>
</tr>
<tr>
<td>Regression diagnostics post-processing</td>
<td>regression__diagnostics</td>
<td>none</td>
<td>Optional</td>
<td>no regression diagnostics</td>
</tr>
</tbody>
</table>

Table 5.22: Specification detail for NL2SOL method dependent controls.

### 5.5.2 NLSSOL Method

NLSSOL is available as `nlssol\_sqp` and supports unconstrained, bound-constrained, and generally-constrained problems. It exploits the structure of a least squares objective function through the periodic use of Gauss-Newton Hessian approximations to accelerate the SQP algorithm. DAKOTA provides access to the NLSSOL library through the `NLSSOL\_LeastSq` class. The method independent and method dependent controls are identical to those of NPSOL as described in NPSOL method independent controls and NPSOL method dependent controls.

### 5.5.3 Gauss-Newton Method

The Gauss-Newton algorithm is available as `optpp\_g\_newton` and supports unconstrained, bound-constrained, and generally-constrained problems. The code for the Gauss-Newton approximation (objective function value,
5.6 Surrogate-Based Minimization Methods

In surrogate-based optimization (SBO) and surrogate-based nonlinear least squares (SBNLS), minimization occurs using a set of one or more approximations, defined from a surrogate model, that are built and periodically updated using data from a "truth" model. The surrogate model can be a global data fit (e.g., regression or interpolation of data generated from a design of computer experiments), a multipoint approximation, a local Taylor Series expansion, or a model hierarchy approximation (e.g., a low-fidelity simulation model), whereas the truth model involves a high-fidelity simulation model. The goals of surrogate-based methods are to reduce the total number of truth model simulations and, in the case of global data fit surrogates, to smooth noisy data with an easily navigated analytic function.

5.6.1 Surrogate-Based Local Method

In the surrogate-based local (SBL) method, a trust region approach is used to manage the minimization process to maintain acceptable accuracy between the surrogate model and the truth model (by limiting the range over which the surrogate model is trusted). The process involves a sequence of minimizations performed on the surrogate model and bounded by the trust region. At the end of each approximate minimization, the candidate optimum point is validated using the truth model. If sufficient decrease has been obtained in the truth model, the trust region is re-centered around the candidate optimum point and the trust region will either shrink, expand, or remain the same size depending on the accuracy with which the surrogate model predicted the truth model decrease. If sufficient decrease has not been attained, the trust region center is not updated and the entire trust region shrinks by a user-specified factor. The cycle then repeats with the construction of a new surrogate model, a minimization, and another test for sufficient decrease in the truth model. This cycle continues until convergence is attained.

The surrogate_based_local method must specify an optimization or least squares sub-method either by pointer using approx_method_pointer (e.g., 'NLP1') or by name using approx_method_name (e.g., 'npsol_sqp'). The former identifies a full sub-method specification for the sub-problem minimizer (which allows non-default minimizer settings), whereas the latter supports a streamlined specification (that employs default minimizer settings). For both cases, the surrogate_based_local method specification is responsible for using its model_pointer (see Method Independent Controls) to select a surrogate model (see Surrogate Model Controls). Any model_pointer identified in an approximate sub-method specification is ignored.

In addition to the method independent controls for max_iterations and convergence_tolerance described in Table 5.1, SBL algorithm controls include soft_convergence_limit (a soft convergence control for the SBL iterations which limits the number of consecutive iterations with improvement less than the convergence tolerance) and truth_surrogate_bypass (a flag for bypassing all lower level surrogates when performing truth verifications on a top level surrogate). Table 5.23 summarizes these SBL inputs.
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surrogate-based local method</td>
<td>surrogate_based_local</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Approximate sub-problem minimization method pointer</td>
<td>approx_method_pointer</td>
<td>string</td>
<td>Required (1 of 2 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Approximate sub-problem minimization method name</td>
<td>approx_method_name</td>
<td>string</td>
<td>Required (1 of 2 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Soft convergence limit for SBL iterations</td>
<td>soft_convergence_limit</td>
<td>integer</td>
<td>Optional</td>
<td>5</td>
</tr>
<tr>
<td>Flag for bypassing lower level surrogates in truth verifications</td>
<td>truth_surrogate_bypass</td>
<td>none</td>
<td>Optional</td>
<td>no bypass</td>
</tr>
</tbody>
</table>

Table 5.23: Specification detail for surrogate-based local minimization method

The trust_region optional group specification can be used to specify the initial size of the trust region (using initial_size) relative to the total variable bounds, the minimum size of the trust region (using minimum_size), the contraction factor for the trust region size (using contraction_factor) used when the surrogate model is performing poorly, and the expansion factor for the trust region size (using expansion_factor) used when the surrogate model is performing well. Two additional commands are the trust region size contraction threshold (using contract_threshold) and the trust region size expansion threshold (using expand_threshold). These two commands are related to what is called the trust region ratio, which is the actual decrease in the truth model divided by the predicted decrease in the truth model in the current trust region. The command contract_threshold sets the minimum acceptable value for the trust region ratio, i.e., values below this threshold cause the trust region to shrink for the next SBL iteration. The command expand_threshold determines the trust region value above which the trust region will expand for the next SBL iteration. Table 5.24 summarizes these trust region inputs.

For SBL problems with nonlinear constraints, a number of algorithm formulations exist as described in [Eldred and Dunlavy, 2006] and as summarized in the Advanced Examples section of the Models chapter of the Users Manual [Adams et al., 2010]. First, the "primary" functions (that is, the objective functions or least squares terms) in the approximate subproblem can be selected to be surrogates of the original primary functions (original_primary), a single objective function (single_objective) formed from the primary function surrogates, or either an augmented Lagrangian merit function (augmented_lagrangian_objective) or a Lagrangian merit function (lagrangian_objective) formed from the primary and secondary function surrogates. The former option may imply the use of a nonlinear least squares method, a multiobjective optimization method, or a single objective optimization method to solve the approximate subproblem, depending on the definition of the primary functions. The latter three options all imply the use of a single objective optimization method regardless of primary function definition. Second, the surrogate constraints in the approximate subproblem can be selected to be surrogates of the original constraints (original_constraints) or linearized approximations to the surrogate constraints (linearized_constraints), or constraints can be omitted from the subproblem (no_constraints). Following optimization of the approximate subproblem, the candidate iterate is evaluated using a merit function, which can be selected to be a simple penalty function with penalty ramped by SBL itera-
Table 5.24: Specification detail for trust region controls in surrogate-based local methods

5.6.2 Surrogate-Based Global Method

The surrogate_based_global method differs from the surrogate_based_local method in a few ways. First, surrogate_based_global is not a trust region method. Rather, surrogate_based_global works in an iterative scheme where optimization is performed on a global surrogate using the same bounds during each iteration. In one iteration, the optimal solutions of the surrogate model are found, and then a selected set of these optimal surrogate solutions are passed to the next iteration. At the next iteration, these surrogate points are evaluated with the "truth" model, and then these points are added back to the set of points upon which the next surrogate is constructed. In this way, the optimization acts on a more accurate surrogate
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
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</thead>
<tbody>
<tr>
<td>Approximate subproblem formulation</td>
<td>approx_-subproblem</td>
<td>original_-_primary</td>
<td>single_-objective</td>
<td>augmented_-lagrangian_-objective</td>
</tr>
<tr>
<td>SBL merit function</td>
<td>merit_-function</td>
<td>penalty_-merit</td>
<td>adaptive_-penalty_-merit</td>
<td>lagrangian_-merit</td>
</tr>
<tr>
<td>SBL iterate acceptance logic</td>
<td>acceptance_-logic</td>
<td>tr_ratio</td>
<td>filter</td>
<td>Optional group</td>
</tr>
<tr>
<td>SBL constraint relaxation method for infeasible iterates</td>
<td>constraint_-relax</td>
<td>homotopy</td>
<td>Optional group</td>
<td>no relaxation</td>
</tr>
</tbody>
</table>

Table 5.25: Specification detail for constraint management in surrogate-based local methods
during each iteration, presumably driving to optimality quickly. This approach has no guarantee of convergence. It was originally designed for MOGA (a multi-objective genetic algorithm). Since genetic algorithms often need thousands or tens of thousands of points to produce optimal or near-optimal solutions, the use of surrogates can be helpful for reducing the truth model evaluations. Instead of creating one set of surrogates for the individual objectives and running the optimization algorithm on the surrogate once, the idea is to select points along the (surrogate) Pareto frontier, which can be used to supplement the existing points. In this way, one does not need to use many points initially to get a very accurate surrogate. The surrogate becomes more accurate as the iterations progress. Note that the user has the option of appending the optimal points from the surrogate model to the current set of truth points or using the optimal points from the surrogate model to replace the optimal set of points from the previous iteration. Although appending to the set is the default behavior, at this time we strongly recommend using the option replace_points because it appears to be more accurate and robust. Finally, the number of best solutions that will be passed from one iteration to another is governed by the iterator control final_solutions. If this is not specified, the surrogate-based global method will take all of the solutions available (e.g. all of the solutions in the Pareto front).

As for the surrogate_based_local method, the surrogate_based_global specification must identify a sub-method using either approx_method_pointer or approx_method_name and must identify a surrogate model (see Surrogate Model Controls) using its model_pointer (see Method Independent Controls). The only other algorithm control at this time is the method independent control for max_iterations described in Table 5.1. Table 5.26 summarizes the method dependent surrogate based global inputs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surrogate-based global method</td>
<td>surrogate_based_global</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Approximate sub-problem minimization method pointer</td>
<td>approx_method_pointer</td>
<td>string</td>
<td>Required (1 of 2 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Approximate sub-problem minimization method name</td>
<td>approx_method_name</td>
<td>string</td>
<td>Required (1 of 2 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Replace points used in surrogate construction with best points from previous iteration</td>
<td>replace_points</td>
<td>none</td>
<td>Optional</td>
<td>Points appended, not replaced</td>
</tr>
</tbody>
</table>

Table 5.26: Specification detail for the surrogate-based global method

We have two cautionary notes before using the surrogate-based global method:

- One might first try a single minimization method coupled with a surrogate model prior to using the surrogate-based global method. This is essentially equivalent to setting max_iterations to 1 and will allow one to get a sense of what surrogate types are the most accurate to use for the problem. (Also note that one can specify that surrogates be built for all primary functions and constraints or for only a subset of these functions and constraints. This allows one to use a “truth” model directly for some of the response functions, perhaps due to them being much less expensive than other functions. This is outlined in Surrogate Model Controls.)

- We initially recommend a small number of maximum iterations, such as 3-5, to get a sense of how the
optimization is evolving as the surrogate gets updated. If it appears to be changing significantly, then a larger number (used in combination with restart) may be needed.

5.6.3 Efficient Global Method

The Efficient Global Optimization (EGO) method was first developed by Jones, Schonlau, and Welch [Jones et al., 1998]. In EGO, a stochastic response surface approximation for the objective function is developed based on some sample points from the "true" simulation. The particular response surface used is a Gaussian process (GP). The GP allows one to calculate the prediction at a new input location as well as the uncertainty associated with that prediction. The key idea in EGO is to maximize the Expected Improvement Function (EIF). The EIF is used to select the location at which a new training point should be added to the Gaussian process model by maximizing the amount of improvement in the objective function that can be expected by adding that point. A point could be expected to produce an improvement in the objective function if its predicted value is better than the current best solution, or if the uncertainty in its prediction is such that the probability of it producing a better solution is high. Because the uncertainty is higher in regions of the design space with few observations, this provides a balance between exploiting areas of the design space that predict good solutions, and exploring areas where more information is needed. EGO trades off this "exploitation vs. exploration." The general procedure for these EGO-type methods is:

- Build an initial Gaussian process model of the objective function
- Find the point that maximizes the EIF. If the EIF value at this point is sufficiently small, stop.
- Evaluate the objective function at the point where the EIF is maximized. Update the Gaussian process model using this new point. Return to the previous step.

Note that several major differences exist between our implementation and that of [Jones et al., 1998]. First, rather than using a branch and bound method to find the point which maximizes the EIF, we use the DIRECT global optimization method (see DIRECT and NCSU Methods). Second, we support both global optimization and global nonlinear least squares as well as general nonlinear constraints through abstraction and subproblem recasting within the SurrBasedMinimizer and EffGlobalMinimizer classes.

The efficient global method is in prototype form. Currently, we do not expose any specification controls for the underlying Gaussian process model used or for the optimization of the expected improvement function (which is currently performed by the NCSU DIRECT algorithm using its internal defaults). Future releases may allow more specification detail. The efficient global algorithm is specified by the keyword efficient_global along with an optional seed specification, as shown in in Table 5.27 below.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
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</thead>
<tbody>
<tr>
<td>Efficient global method</td>
<td>efficient_-global</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Random seed</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>Time based seed: nonrepeatable</td>
</tr>
</tbody>
</table>

Table 5.27: Specification detail for the efficient global method
5.7 Uncertainty Quantification Methods

DAKOTA has several methods for propagating uncertainty. Aleatory uncertainty refers to inherent variability, irreducible uncertainty, or randomness, and is addressed with the probabilistic methods described in Aleatory Uncertainty Quantification Methods. Epistemic uncertainty refers to subjective uncertainty, reducible uncertainty, model form uncertainty, or uncertainty due to lack of knowledge, and is addressed using the non-probabilistic approaches described in Epistemic Uncertainty Quantification Methods. In general, we refer to both classes of uncertainty quantification methods in DAKOTA as nondeterministic methods. In the descriptions below, we described the issues and specification controls that are common to both aleatory and epistemic uncertainty quantification.

DAKOTA’s nondeterministic methods do not make use of many method independent controls. Only the x_taylor_mpp, u_taylor_mpp, x_two_point, and u_two_point methods within local_reliability use method independent convergence controls (see Local reliability methods). As such, the nondeterministic branch documentation which follows is primarily limited to the method dependent controls for the sampling, reliability, stochastic expansion, and epistemic methods.

With a few exceptions (global_reliability, importance, local_evidence, and global_evidence do not support mappings involving reliability_levels, and local_interval_est and global_interval_est do not support any level mappings), each of these techniques supports response_levels, probability_levels, reliability_levels, and gen_reliability_levels specifications along with optional num_response_levels, num_probability_levels, num_reliability_levels, and num_gen_reliability_levels keys. The keys define the distribution of the levels among the different response functions. For example, the following specification:

```
num_response_levels = 2 4 3
response_levels = 1. 2. .1 .2 .3 .4 10. 20. 30.
```

would assign the first two response levels (1., 2.) to response function 1, the next four response levels (.1, .2, .3, .4) to response function 2, and the final three response levels (10., 20., 30.) to response function 3. If the num_response_levels key were omitted from this example, then the response levels would be evenly distributed among the response functions (three levels each in this case).

The response_levels specification provides the target response values for generating probabilities, reliabilities, or generalized reliabilities (forward mapping). The selection among these possible results for the forward mapping is performed with the compute keyword followed by either probabilities, reliabilities, or gen_reliabilities. Conversely, the probability_levels, reliability_levels, and gen_reliability_levels specifications provide target levels for which response values will be computed (inverse mapping). Specifications of response_levels, probability_levels, reliability_levels, and gen_reliability_levels may be combined within the calculations for each response function. The mapping results (probabilities, reliabilities, or generalized reliabilities for the forward mapping and response values for the inverse mapping) define the final statistics of the nondeterministic analysis that can be accessed for use at a higher level (via the primary and secondary mapping matrices for nested models; see Nested Model Controls).

Sets of response-probability pairs computed with the forward/inverse mappings define either a cumulative distribution function (CDF) or a complementary cumulative distribution function (CCDF) for each response function. In the case of evidence-based epistemic methods, this is generalized to define either cumulative belief and plausibility functions (CBF and CPF) or complementary cumulative belief and plausibility functions (CCBF and CCPF) for each response function, where a forward mapping involves computing the belief and plausibility probability level for a specified response level and an inverse mapping involves computing the belief and plausibility response level for either a specified probability level or a specified generalized reliability.
level (two results for each level mapping in the evidence-based epistemic case, instead of the one result for each level mapping in the aleatory case). The selection of a CDF/CFB/CPF or CCDF/CCBF/CCPF can be performed with the `distribution` keyword followed by either `cumulative` for the CDF/CFB/CPF option or `complementary` for the CCDF/CCBF/CCPF option. This selection also defines the sign of the reliability or generalized reliability indices. Table 5.28 provides the specification detail for the forward/inverse mappings used by each of the nondeterministic analysis methods.

Different nondeterministic methods have differing support for uncertain variable distributions. Table 5.29 summarizes the uncertain variables that are available for use by the different methods, where a "-" indicates that the distribution is not supported by the method, a "U" means the uncertain input variables of this type must be uncorrelated, a "C" denotes that correlations are supported involving uncertain input variables of this type. Additional notes include:

- we have three variants for stochastic expansions (SE), listed as Wiener, Askey, and Extended, which draw from different sets of basis polynomials. The term stochastic expansion indicates polynomial chaos and stochastic collocation collectively. Refer to Polynomial chaos expansion method and Stochastic collocation method for additional information on these three options.

- methods supporting the epistemic interval distributions have differing approaches: sampling and the `lhs` option of `global_interval_est` model the interval basic probability assignments (BPAs) as continuous histogram bin distributions for purposes of generating samples; `local_interval_est` and the `ego` option of `global_interval_est` ignore the BPA details and models these variables as simple bounded regions defined by the cell extremes; and `local_evidence` and `global_evidence` model the interval specifications as true BPAs.

### 5.7.1 Aleatory Uncertainty Quantification Methods

Aleatory uncertainty is also known as inherent variability, irreducible uncertainty, or randomness. An example of aleatory uncertainty is the distribution of height in a population, as it is characterized by the availability of sufficient data to accurately model the form of the variation. For this reason, aleatory uncertainty is typically modeled using probabilistic approaches through the specification of probability distributions to represent the uncertain input variables and the propagation of this uncertainty using probability theory. The probabilistic approaches supported in DAKOTA include sampling, local and global reliability, polynomial chaos, and stochastic collocation, which may be used to propagate random variables described by Normal Distribution, Lognormal Distribution, Uniform Distribution, Loguniform Distribution, Triangular Distribution, Exponential Distribution, Beta Distribution, Gamma Distribution, Gumbel Distribution, Frechet Distribution, Weibull Distribution, Histogram Bin Distribution, Poisson Distribution, Binomial Distribution, Negative Binomial Distribution, Geometric Distribution, Hypergeometric Distribution, and/or Histogram Point Distribution.

#### Nondeterministic sampling method

The nondeterministic sampling method is selected using the `sampling` specification. This method draws samples from the specified uncertain variable probability distributions and propagates them through the model to obtain statistics on the output response functions of interest. DAKOTA provides access to nondeterministic sampling methods through the combination of the `NonDSampling` base class and the `NonDLHSSampling` derived class.

CDF/CCDF probabilities are calculated for specified response levels using a simple binning approach. Response levels are calculated for specified CDF/CCDF probabilities and generalized reliabilities by indexing into a sorted samples array (the response levels computed are not interpolated and will correspond to one of the sampled values). CDF/CCDF reliabilities are calculated for specified response levels by computing the number of sample...
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
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<tbody>
<tr>
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<td>cumulative</td>
<td>Optional group</td>
<td>cumulative (CDF)</td>
</tr>
<tr>
<td>Response levels</td>
<td>response_levels</td>
<td>list of reals</td>
<td>Optional group</td>
<td>No CDF/CCDF probabilities/reliabilities to compute</td>
</tr>
<tr>
<td>Number of response levels</td>
<td>num_response_levels</td>
<td>list of integers</td>
<td>Optional</td>
<td>response_levels evenly distributed among response functions</td>
</tr>
<tr>
<td>Target statistics for response levels</td>
<td>compute</td>
<td>probabilities</td>
<td>Optional</td>
<td>probabilities</td>
</tr>
<tr>
<td>Probability levels</td>
<td>probability_levels</td>
<td>list of reals</td>
<td>Optional group</td>
<td>No CDF/CCDF response levels to compute</td>
</tr>
<tr>
<td>Number of probability levels</td>
<td>num_probability_levels</td>
<td>list of integers</td>
<td>Optional</td>
<td>probability_levels evenly distributed among response functions</td>
</tr>
<tr>
<td>Reliability levels</td>
<td>reliability_levels</td>
<td>list of reals</td>
<td>Optional group</td>
<td>No CDF/CCDF response levels to compute</td>
</tr>
<tr>
<td>Number of reliability levels</td>
<td>num_reliability_levels</td>
<td>list of integers</td>
<td>Optional</td>
<td>reliability_levels evenly distributed among response functions</td>
</tr>
<tr>
<td>Generalized reliability levels</td>
<td>gen_reliability_levels</td>
<td>list of reals</td>
<td>Optional group</td>
<td>No CDF/CCDF response levels to compute</td>
</tr>
<tr>
<td>Number of generalized reliability levels</td>
<td>num_gen_reliability_levels</td>
<td>list of integers</td>
<td>Optional</td>
<td>gen_reliability_levels evenly distributed among response functions</td>
</tr>
</tbody>
</table>

Table 5.28: Specification detail for forward/inverse level mappings
<table>
<thead>
<tr>
<th>Distribution Type</th>
<th>Sampling</th>
<th>Local Reliability</th>
<th>Global Reliability</th>
<th>Wiener SE</th>
<th>Askey SE</th>
<th>Extended SE</th>
<th>Local Interval</th>
<th>Global Interval</th>
<th>Local Evidence</th>
<th>Global Evidence</th>
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<tr>
<td>Bounded Normal</td>
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<td>U</td>
<td>U</td>
<td>U</td>
<td>U</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Log-normal</td>
<td>C</td>
<td>C</td>
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<td>C</td>
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Table 5.29: Summary of Distribution Types supported by Nondeterministic Methods
standard deviations separating the sample mean from the response level. Response levels are calculated for specified CDF/CCDF reliabilities by projecting out the prescribed number of sample standard deviations from the sample mean.

The seed integer specification specifies the seed for the random number generator which is used to make sampling studies repeatable, and rng specifies which random number generator is used. The fixed_seed flag is relevant if multiple sampling sets will be generated during the course of a strategy (e.g., surrogate-based optimization, optimization under uncertainty). Specifying this flag results in the reuse of the same seed value for each of these multiple sampling sets, which can be important for reducing variability in the sampling results. However, this behavior is not the default as the repetition of the same sampling pattern can result in a modeling weakness that an optimizer could potentially exploit (resulting in actual reliabilities that are lower than the estimated reliabilities). In either case (fixed_seed or not), the study is repeatable if the user specifies a seed and the study is random is the user omits a seed specification.

The number of samples to be evaluated is selected with the samples integer specification. The algorithm used to generate the samples can be specified using sample_type followed by either random, for pure random Monte Carlo sampling, or lhs, for Latin Hypercube sampling.

If the user wants to increment a particular set of samples with more samples to get better estimates of mean, variance, and percentiles, one can select incremental_random or incremental_lhs as the sample_type. Note that a preliminary sample of size N must have already been performed, and a dakota.rst restart file must be available from this original sample. For example, say a user performs an initial study using lhs as the sample_type, and generates 50 samples. If the user creates a new input file where samples is now specified to be 100, the sample_type is defined to be incremental_lhs or incremental_random, and previous_samples is specified to be 50, the user will get 50 new LHS samples which maintain both the correlation and stratification of the original LHS sample. The N new samples will be combined with the N original samples to generate a combined sample of size 2N. The syntax for running the second sample set is: dakota -i input2.in -r dakota.rst, where input2.in is the file which specifies incremental sampling. Note that the number of samples in the second set MUST currently be 2 times the number of previous samples, although incremental sampling based on any power of two may be supported in future releases.

The nondeterministic sampling method also supports a design of experiments mode through the all_variables flag. Normally, sampling generates samples only for the uncertain variables, and treats any design or state variables as constants. The all_variables flag alters this behavior by instructing the sampling algorithm to treat any continuous design or continuous state variables as parameters with uniform probability distributions between their upper and lower bounds. Samples are then generated over all of the continuous variables (design, uncertain, and state) in the variables specification. This is similar to the behavior of the design of experiments methods described in Design of Computer Experiments Methods, since they will also generate samples over all continuous design, uncertain, and state variables in the variables specification. However, the design of experiments methods will treat all variables as being uniformly distributed between their upper and lower bounds, whereas the sampling method will sample the uncertain variables within their specified probability distributions.

Finally, the nondeterministic sampling method supports two types of sensitivity analysis. In this context of sampling, we take sensitivity analysis to be global, not local as when calculating derivatives of output variables with respect to input variables. Our definition is similar to that of [Saltelli et al., 2004]: ”The study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input.” As a default, DAKOTA provides correlation analyses when running LHS. Correlation tables are printed with the simple, partial, and rank correlations between inputs and outputs. These can be useful to get a quick sense of how correlated the inputs are to each other, and how correlated various outputs are to inputs. In addition, we have the capability to calculate sensitivity indices through variance based decomposition using the keyword variance_based_decomp. Variance based decomposition is a way of using sets of samples to understand how the variance of the
output behaves, with respect to each input variable. A larger value of the sensitivity index, $S_i$, means that the uncertainty in the input variable $i$ has a larger effect on the variance of the output. More details on the calculations and interpretation of the sensitivity indices can be found in [Saltelli et al., 2004] and [Weirs et al., 2010]. Note that variance-based decomposition is extremely computationally intensive since replicated sets of sample values are evaluated. If the user specified a number of samples, $N$, and a number of nondeterministic variables, $M$, variance-based decomposition requires the evaluation of $N \times (M + 2)$ samples. To obtain sensitivity indices that are reasonably accurate, we recommend that $N$, the number of samples, be at least one hundred and preferably several hundred or thousands. Because of the computational cost, variance-based decomposition is turned off as a default. Table 5.30 provides details of the nondeterministic sampling specifications beyond those of Table 5.28.

<table>
<thead>
<tr>
<th>Description</th>
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<td>Optional</td>
<td>minimum required</td>
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<td>integer</td>
<td>Optional</td>
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<td>Optional</td>
<td>randomly generated seed</td>
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<td>Optional</td>
<td>seed not fixed: sampling patterns are variable among multiple runs</td>
</tr>
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<td>Optional</td>
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<td>all_-variables</td>
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<td>Optional</td>
<td>sampling only over uncertain variables</td>
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Table 5.30: Specification detail for nondeterministic sampling method

**Local reliability methods** Local reliability methods are selected using the local_reliability specification and are implemented within the NonDLocalReliability class. These methods compute approximate response function distribution statistics based on specified uncertain variable probability distributions. Each of the local reliability methods can compute forward and inverse mappings involving response, probability, reliability, and generalized reliability levels.

The Mean Value method (MV, also known as MVFOSM in [Haldar and Mahadevan, 2000]) is the simplest,
least-expensive method in that it estimates the response means, response standard deviations, and all CDF/CCDF forward/inverse mappings from a single evaluation of response functions and gradients at the uncertain variable means. This approximation can have acceptable accuracy when the response functions are nearly linear and their distributions are approximately Gaussian, but can have poor accuracy in other situations.

All other reliability methods perform an internal nonlinear optimization to compute a most probable point (MPP) of failure. A sign convention and the distance of the MPP from the origin in the transformed standard normal space ("u-space") define the reliability index, as explained in the section on Reliability Methods in the Uncertainty Quantification chapter of the Users Manual [Adams et al., 2010]. The reliability can then be converted to a probability using either first- or second-order integration, may then be refined using importance sampling, and finally may be converted to a generalized reliability index. The forward reliability analysis algorithm of computing reliabilities/probabilities for specified response levels is called the Reliability Index Approach (RIA), and the inverse reliability analysis algorithm of computing response levels for specified probability levels is called the Performance Measure Approach (PMA). The different RIA/PMA algorithm options are specified using the mpp_search specification which selects among different limit state approximations that can be used to reduce computational expense during the MPP searches. The x_taylor_mean MPP search option performs a single Taylor series approximation in the space of the original uncertain variables ("x-space") centered at the uncertain variable means, searches for the MPP for each response/probability level using this approximation, and performs a validation response evaluation at each predicted MPP. This option is commonly known as the Advanced Mean Value (AMV) method. The u_taylor_mean option is identical to the x_taylor_mean option, except that the approximation is performed in u-space. The x_taylor_mpp approach starts with an x-space Taylor series at the uncertain variable means, but iteratively updates the Taylor series approximation at each MPP prediction until the MPP converges. This option is commonly known as the AMV+ method. The u_taylor_mpp option is identical to the x_taylor_mpp option, except that all approximations are performed in u-space. The order of the Taylor-series approximation is determined by the corresponding responses specification and may be first or second-order. If second-order (methods named AMV and AMV in [Eldred and Bichon, 2006]), the series may employ analytic, finite difference, or quasi Hessians (BFGS or SR1). The x_two_point MPP search option uses an x-space Taylor series approximation at the uncertain variable means for the initial MPP prediction, then utilizes the Two-point Adaptive Nonlinear Approximation (TANA) outlined in [Xu and Grandhi, 1998] for all subsequent MPP predictions. The u_two_point approach is identical to x_two_point, but all the approximations are performed in u-space. The x_taylor_mpp and u_taylor_mpp, x_two_point and u_two_point approaches utilize the max_iterations and convergence_tolerance method independent controls to control the convergence of the MPP iterations (the maximum number of MPP iterations per level is limited by max_iterations, and the MPP iterations are considered converged when \( \| u^{(k+1)} - u^{(k)} \|_2 < \text{convergence_tolerance} \)). And, finally, the no_approx option performs the MPP search on the original response functions without the use of any approximations. The optimization algorithm used to perform these MPP searches can be selected to be either sequential quadratic programming (uses the npsol_sqp optimizer) or nonlinear interior point (uses the optpp_q_newton optimizer) algorithms using the sqp or nip keywords.

In addition to the MPP search specifications, one may select among different integration approaches for computing probabilities at the MPP by using the integration keyword followed by either first_order or second_order. Second-order integration employs the formulation of [Hohenbichler and Rackwitz, 1988] (the approach of [Breitung, 1984] and the correction of [Hong 1999] are also implemented, but are not active). Combining the no_approx option of the MPP search with first- and second-order integrations results in the traditional first- and second-order reliability methods (FORM and SORM). These integration approximations may be subsequently refined using importance sampling. The refinement specification allows the selection of basic importance sampling (import), adaptive importance sampling (adapt_import), or multimodal adaptive importance sampling (mm_adapt_import), along with the specification of number of samples (samples) and random seed (seed). Additional details on these methods are available in [Eldred et al., 2004b] and [Eldred and Bichon, 2006] and in the Uncertainty Quantification Capabilities chapter of the Users Manual [Adams et al.,
Table 5.31 provides details of the local reliability method specifications beyond those of Table 5.28.

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<td>Optional group</td>
<td>randomly generated seed</td>
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Table 5.31: Specification detail for local reliability methods

**Global reliability methods**  Global reliability methods are selected using the `global_reliability` specification and are implemented within the `NonDGlobalReliability` class. These methods do not support forward/inverse mappings involving `reliability_levels`, since they never form a reliability index based on distance in u-space. Rather they use a Gaussian process model to form an approximation to the limit state (based either in x-space via the `x_gaussian_process` specification or in u-space via the `u_gaussian_process` specification), followed by probability estimation based on multimodal adaptive importance sampling (see [Bichon et al., 2007]). These probability estimates may then be transformed into generalized reliability levels if desired. At this time, inverse reliability analysis (mapping probability or generalized reliability levels into response levels) is not yet operational, although it may be supported in future releases. The Gaussian process model approximation to the limit state is formed over the uncertain variables by default, but may be extended to also capture the effect of design and state variables via the `all_variables` flag.

Table 5.32 provides details of the global reliability method specifications beyond those of Table 5.28.

**Polynomial chaos expansion method**  The polynomial chaos expansion (PCE) is a general framework for the approximate representation of random response functions in terms of finite-dimensional series expansions in standardized random variables.
Table 5.32: Specification detail for global reliability methods

\[ R = \alpha_i^P i=0 \Psi_i(\xi) \]

where \( \alpha_i \) is a deterministic coefficient, \( \Psi_i \) is a multidimensional orthogonal polynomial and \( \xi \) is a vector of standardized random variables. An important distinguishing feature of the methodology is that the functional relationship between random inputs and outputs is captured, not merely the output statistics as in the case of many nondeterministic methodologies. DAKOTA provides access to PCE methods through the NonDPolynomial-Chaos class. Refer to the Uncertainty Quantification Capabilities chapter of the Users Manual [Adams et al., 2010] for additional information on the PCE algorithm.

To select the basis \( \Psi_i \) of the expansion, three approaches may be employed, as previously shown in Table 5.29: Wiener, Askey, and Extended. The Wiener option uses a Hermite orthogonal polynomial basis for all random variables and employs the same nonlinear variable transformation as the local and global reliability methods (and therefore has the same variable support). The Askey option, however, employs an extended basis of Hermite, Legendre, Laguerre, Jacobi, and generalized Laguerre orthogonal polynomials. The Extended option avoids the use of any nonlinear variable transformations by augmenting the Askey approach with numerically-generated orthogonal polynomials for non-Askey probability density functions. The selection of Wiener versus Askey versus Extended is partially automated and partially under the user’s control. The Extended option is the default and supports only Gaussian correlations (see Table 5.29). This default can be overridden by the user by supplying the keyword \texttt{askey} to request restriction to the use of Askey bases only or by supplying the keyword \texttt{wiener} to request restriction to the use of exclusively Hermite bases. If needed to support prescribed correlations (not under user control), the Extended and Askey options will fall back to the Wiener option \textit{on a per variable basis}. If the prescribed correlations are also unsupported by Wiener expansions, then DAKOTA will exit with an error. Additional details include:

- Askey polynomial selections include Hermite for normal (optimal) as well as bounded normal, lognormal, bounded lognormal, gumbel, frechet, and weibull (sub-optimal); Legendre for uniform (optimal) as well as loguniform, triangular, and bin-based histogram (sub-optimal); Laguerre for exponential (optimal); Jacobi for beta (optimal); and generalized Laguerre for gamma (optimal).

- Extended polynomial selections replace each of the sub-optimal Askey basis selections with numerically-generated polynomials that are orthogonal to the prescribed probability density functions (for bounded
normal, lognormal, bounded lognormal, loguniform, triangular, gumbel, frechet, weibull, and bin-based histogram).

To obtain the coefficients $\alpha_i$ of the expansion, six options are provided:

1. multidimensional integration by a tensor-product of Gaussian quadrature rules (specified with quadrature_order, where the order per variable may be anisotropic). The default rule selection is to employ non_nested Gauss rules including Gauss-Hermite (for normals or transformed normals), Gauss-Legendre (for uniforms or transformed uniforms), Gauss-Jacobi (for betas), Gauss-Laguerre (for exponentials), generalized Gauss-Laguerre (for gammas), and numerically-generated Gauss rules (for other distributions when using an Extended basis). For the case of p_refinement or the case of an explicit nested override, Gauss-Hermite rules are replaced with Genz-Keister nested rules and Gauss-Legendre rules are replaced with Gauss-Patterson nested rules, both of which exchange lower integrand precision for greater point reuse. To synchronize with tensor-product integration, a tensor-product expansion is used, where the order $p_i$ of the expansion in each dimension is selected to be half of the integrand precision available from the rule in use, rounded down. In the case of non-nested Gauss rules with integrand precision $2^{m_i} - 1$, $p_i$ is one less than the quadrature order $m_i$ in each dimension. The total number of terms, $N$, in a tensor-product expansion involving $n$ uncertain input variables is

$$N = 1 + P = \prod_{i=1}^{n} (p_i + 1)$$

2. multidimensional integration by the Smolyak sparse grid method (specified with sparse_grid_level and, optionally, dimension_preference). The underlying one-dimensional integration rules are the same as for the tensor-product quadrature case; however, the default rule selection is nested for sparse grids (Genz-Keister for normals/transformed normals and Gauss-Patterson for uniforms/transformed uniforms). This default can be overridden with an explicit non_nested specification (resulting in Gauss-Hermite for normals/transformed normals and Gauss-Legendre for uniforms/transformed uniforms). By specifying a dimension_preference, where higher preference leads to higher order polynomial resolution, the sparse grid may be rendered anisotropic (refer to the PCE description in the User’s Manual for the anisotropic index set constraint definition). For PCE with either isotropic or anisotropic sparse grids, a summation of tensor-product expansions is used, where each anisotropic tensor-product quadrature rule underlying the sparse grid construction results in its own anisotropic tensor-product expansion as described in case 1. These anisotropic tensor-product expansions are summed into a sparse PCE using the standard Smolyak summation (again, refer to the User’s Manual for additional details).

3. multidimensional integration by Stroud cubature rules [Stroud, 1971] and extensions [Xiu, 2008], as specified with cubature_integrand. A total-order expansion is used, where the isotropic order $p$ of the expansion is half of the integrand order, rounded down. The total number of terms $N$ for an isotropic total-order expansion of order $p$ over $n$ variables is given by

$$N = 1 + P = 1 + \sum_{s=1}^{p} \frac{1}{s!} \prod_{r=0}^{s-1} (n + r) = \frac{(n + p)!}{n!p!}$$

Since the maximum integrand order is currently five for normal and uniform and two for all other types, at most second- and first-order expansions, respectively, will be used. As a result, cubature is primarily useful for global sensitivity analysis, where the Sobol’ indices will provide main effects and, at most, two-way interactions. In addition, the random variable set must be independent and identically distributed (iid), so the use of askey or wiener transformations may be required to create iid variable sets in the transformed...
space (as well as to allow usage of the higher order cubature rules for normal and uniform). Note that global sensitivity analysis often assumes uniform bounded regions, rather than precise probability distributions, so the iid restriction would not be problematic in that case.

4. multidimensional integration by random sampling (specified with `expansion_samples`). In this case, the expansion order $p$ cannot be inferred from the numerical integration specification and it is necessary to provide either `expansion_order` or `expansion_terms` to specify either $p$ or $N$, respectively, for a total-order expansion, where the latter specification allows the use of a partial-order expansion (truncation of a complete order expansion, while supported, is not generally recommended).

5. linear regression (specified with either `collocation_points` or `collocation_ratio`). A total-order expansion is used and must be specified using either `expansion_order` or `expansion_terms` as described in the previous option. Given $p$ or $N$, the total number of collocation points (including any sample reuse) must be at least $N$, and an oversampling is generally advisable. To more easily satisfy this requirement (i.e., to avoid requiring the user to calculate $N$ from $n$ and $p$), `collocation_ratio` allows for specification of a constant oversampling factor applied to $N$ (e.g., `collocation_ratio = 2` for factor of 2 oversampling). The `use_derivatives` flag informs the regression approach to include derivative matching equations (limited to gradients at present) in the least squares solutions, enabling the use of fewer collocation points for a given expansion order and dimension (number of points required becomes $\frac{2N}{p+1}$, where $r$ is the `collocation_ratio`). When admissible, a constrained least squares approach is employed in which response values are first reproduced exactly and error in reproducing response derivatives is minimized.

6. coefficient import from a file (specified with `expansion_import_file`). A total-order expansion is assumed and must be specified using either `expansion_order` or `expansion_terms`.

If $n$ is small (e.g., two or three), then tensor-product Gaussian quadrature is quite effective and can be the preferred choice. For moderate $n$ (e.g., five), tensor-product quadrature quickly becomes too expensive and the sparse grid and point collocation approaches are preferred. For large $n$ (e.g., more than ten), point collocation may begin to suffer from ill-conditioning and sparse grids are generally recommended. Random sampling for coefficient estimation is generally not recommended, although it does hold the advantage that the simulation budget is more flexible than that required by the other approaches. For incremental studies, approaches 4 and 5 support reuse of previous samples through the `incremental_lhs` (refer to Nondeterministic sampling method for description of incremental LHS) and `reuse_samples` (refer to Global approximations for description of the "all" option of sample reuse) specifications, respectively. The `sample_refinement` specification is similar to that of Local reliability methods, with the difference that the number of refinement samples is not under the user’s control. This option allows for refinement of probability and generalized reliability results using importance sampling. As for Nondeterministic sampling method and Global reliability methods, the `all_variables` flag can be used to form expansions over all continuous variables, rather than just the default aleatory uncertain variables. For continuous design, continuous state, and epistemic interval variables included in `all_variables` mode, Legendre chaos bases are used to model the bounded intervals for these variables. However, these variables are not assumed to have any particular probability distribution, only that they are independent variables. Moreover, when probability integrals are evaluated, only the aleatory random variable domain is integrated, leaving behind a polynomial relationship between the statistics and the remaining design/state/epistemic variables.

In the quadrature and sparse grid cases, growth rates for nested and non-nested rules are synchronized for consistency. For a non-nested Gauss rule used within a sparse grid, linear one-dimensional growth rules of $m = 2l + 1$ are used to enforce odd quadrature orders, where $l$ is the grid level and $m$ is the number of points in the rule. The precision of this Gauss rule is then $i = 2m - 1 = 4l + 1$. For nested rules, order growth with level is typically exponential; however, we restrict the number of points to be the lowest order rule that is available that meets the one-dimensional precision requirement implied by either a level $l$ for a sparse grid ($i = 4l + 1$) or an order $m$ for
a tensor grid \((i = 2m - 1)\). The sole exception to this logic is the \texttt{p_refinement} adaptive generalized case described below, since the ability to evolve the index sets of a sparse grid in an unstructured manner removes the need to restrict the exponential growth of nested rules.

The \texttt{p_refinement} keyword specifies the usage of automated polynomial order refinement, which can be either \texttt{uniform} or \texttt{adaptive}. This is currently supported for the tensor-product quadrature and Smolyak sparse grid options, and makes use of the \texttt{max_iterations} and \texttt{convergence_tolerance} method independent controls (see Table 5.1). The former control limits the number of refinement iterations, and the latter control terminates refinement when the two-norm of the change in the response covariance matrix (or, in goal-oriented approaches, the two-norm of change in the statistical quantities of interest (QOI)) falls below the tolerance. The \texttt{adaptive} case can be further specified to utilize \texttt{sobol}, \texttt{decay}, or \texttt{generalized} refinement controls. The former two adaptive cases employ anisotropic tensor/sparse grids in which the anisotropic dimension preference (leading to anisotropic integrations/expansions with differing refinement levels for different random dimensions) is determined from either total Sobol’ indices from variance-based decomposition (\texttt{sobol} case: high indices result in high dimension preference) or from spectral coefficient decay rates from Richardson extrapolation (\texttt{decay} case: low decay rates result in high dimension preference). In these two cases as well as the \texttt{uniform} refinement case, the \texttt{quadrature_order} or \texttt{sparse_grid_level} are ramped by one on each refinement iteration until either of the two convergence controls is satisfied. Finally, the \texttt{generalized adaptive} case is the default adaptive approach; it refers to the generalized sparse grid algorithm, in which candidate index sets are evaluated for their impact on the statistical QOI, the most influential sets are selected and used to generate additional candidates, and the index set frontier of a sparse grid is evolved in an unstructured and goal-oriented manner (refer to User’s Manual PCE descriptions for additional specifics).

Additional specifications include the level mappings described in Uncertainty Quantification Methods and the \texttt{sample_type}, \texttt{samples}, \texttt{seed}, \texttt{fixed_seed}, and \texttt{rng} specifications described in Nondeterministic sampling method, where the \texttt{sample_type} options are restricted to \texttt{random} and \texttt{lhs}. Each of these sampling specifications refer to sampling on the PCE approximation for the purposes of generating approximate statistics, which should be distinguished from simulation sampling for generating the chaos coefficients as described in options 4 and 5 above. The \texttt{variance_based_decomp} and \texttt{drop_tolerance} are also the same as those described in Nondeterministic sampling method, but since PCE outputs main, interaction, and total effects by default, the \texttt{univariate_effects} option has been added to allow suppression of the interaction effects since the output volume of these results can be prohibitive for high dimensional problems. Table 5.33 provides details of the polynomial chaos expansion specifications beyond those of Table 5.28.

**Stochastic collocation method** The stochastic collocation (SC) method is very similar to the PCE method described above, with the key difference that the orthogonal polynomial basis functions are replaced with Lagrange polynomial interpolants. The expansion takes the form

\[
R = \sum_{i=1}^{N_p} r_i L_i(\xi)
\]

where \(N_p\) is the total number of collocation points, \(r_i\) is a response value at the \(i^{th}\) collocation point, \(L_i\) is the \(i^{th}\) multidimensional Lagrange interpolation polynomial, and \(\xi\) is a vector of standardized random variables. The \(i^{th}\) Lagrange interpolation polynomial assumes the value of 1 at the \(i^{th}\) collocation point and 0 at all other collocation points. Thus, in PCE, one forms coefficients for known orthogonal polynomial basis functions, whereas SC forms multidimensional interpolation functions for known coefficients. DAKOTA provides access to SC methods through the \texttt{NonDSStochCollocation} class. Refer to the Uncertainty Quantification Capabilities chapter of the Users Manual [Adams et al., 2010] for additional information on the SC algorithm.

To form the multidimensional interpolants \(L_i\) of the expansion, two options are provided:
### 5.7 Uncertainty Quantification Methods

<table>
<thead>
<tr>
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<td>wiener</td>
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<td>Optional</td>
</tr>
<tr>
<td>Automated polynomial order refinement</td>
<td>p_refinement</td>
<td>uniform</td>
<td>adaptive</td>
<td>Optional group</td>
</tr>
<tr>
<td>Adaptive refinement control</td>
<td>sobol</td>
<td>decay</td>
<td>generalized</td>
<td>none</td>
</tr>
<tr>
<td>Quadrature order for PCE coefficient estimation</td>
<td>quadrature_<code>order</code></td>
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<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Cubature integrand order for PCE coefficient estimation</td>
<td>cubature_<code>integrand</code></td>
<td>integer (1, 2, 3, or 5 for normal or uniform; 1 or 2 for exponential, beta, or gamma, 2 for all other distribution types)</td>
<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Sparse grid level for PCE coefficient estimation</td>
<td>sparse_<code>grid_level</code></td>
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<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Sparse grid dimension preference</td>
<td>dimension_<code>preference</code></td>
<td>list of reals</td>
<td>Optional</td>
<td>isotropic sparse grid</td>
</tr>
<tr>
<td>Integration rule point nesting for quadrature_<code>order</code> or sparse_<code>grid_level</code></td>
<td>nested</td>
<td>non_nested</td>
<td>None</td>
<td>Optional</td>
</tr>
<tr>
<td>Number of simulation samples for PCE coefficient estimation</td>
<td>expansion_<code>samples</code></td>
<td>integer</td>
<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Incremental LHS flag for PCE coefficient estimation by expansion_<code>samples</code></td>
<td>incremental_<code>lhs</code></td>
<td>none</td>
<td>Optional</td>
<td>coefficient estimation does not reuse previous samples</td>
</tr>
<tr>
<td>Number of collocation points for PCE coefficient estimation</td>
<td>collocation_<code>points</code></td>
<td>integer</td>
<td>Required (1 of 7 selections)</td>
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</tr>
<tr>
<td>Collocation point ratio for PCE coefficient estimation</td>
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<tr>
<td>Derivative usage flag for PCE coefficient estimation</td>
<td>use_<code>derivatives</code></td>
<td>none</td>
<td>Optional</td>
<td>coefficient estimation by function values</td>
</tr>
</tbody>
</table>
1. interpolation on a tensor-product of Gaussian quadrature points (specified with `quadrature_order`, where the order per variable may be anisotropic). As for PCE, non-nested Gauss rules are employed by default, although the presence of `p_refinement` or an explicit `nested` override will result in the use of nested rules for normal or uniform variables after any variable transformations have been applied.

1. interpolation on a Smolyak sparse grid (specified with `sparse_grid_level` and, optionally, `dimension_preference`) defined from Gaussian rules. With the specification of `dimension_preference`, the sparse grid can be rendered anisotropic. As for sparse PCE, nested rules are employed unless specifically overridden with the `non_nested` option.

As for Polynomial chaos expansion method, the orthogonal polynomials used in defining the Gauss points that make up the interpolation grid are governed by one of three options: Wiener, Askey, or Extended. The Wiener option uses interpolation points from Gauss-Hermite (non-nested) or Genz-Keister (nested) integration rules for all random variables and employs the same nonlinear variable transformation as the local and global reliability methods (and therefore has the same variable support). The Askey option, however, employs interpolation points from Gauss-Hermite (Genz-Keister if nested), Gauss-Legendre (Gauss-Patterson if nested), Gauss-Laguerre, Gauss-Jacobi, and generalized Gauss-Laguerre quadrature. The Extended option avoids the use of any nonlinear variable transformations by augmenting the Askey approach with Gauss points from numerically-generated orthogonal polynomials for non-Askey probability density functions. As for PCE, the Wiener/Askey/Extended selection defaults to Extended, can be overridden by the user using the keywords `askey` or `wiener`, and automatically falls back from Extended/Askey to Wiener on a per variable basis as needed to support prescribed correlations.

If \( n \) is small, then tensor-product Gaussian quadrature is again the preferred choice. For larger \( n \), tensor-product quadrature quickly becomes too expensive and the sparse grid approach is preferred. For self-consistency in growth rates, nested rules employ restricted exponential growth for consistency with the linear growth used for non-nested Gauss rules (integrand precision \( i = 4l + 1 \) for sparse grid level and \( i = 2m - 1 \) for tensor grid order), with the exception of the `p_refinement adaptive generalized` case. Similar to the approach described previously in Polynomial chaos expansion method, the `all_variables` flag can be used to expand the dimensionality of the interpolation to include continuous design and state variables and epistemic uncertain variables, in addition to the default aleatory uncertain variables. Interpolation points for these dimensions are based on Gauss-Legendre rules if non-nested and Gauss-Patterson rules if nested. Again, when probability integrals are evaluated, only the aleatory random variable domain is integrated, leaving behind a polynomial relationship between the statistics and the remaining design/state/epistemic variables.

The `p_refinement` keyword can again be specified as either `uniform` or `adaptive`, and the adaptive case may be `sobol` or `generalized` (`decay not supported`). These automated refinement approaches makes use of the `max_iterations` and `convergence_tolerance` iteration controls (see Table 5.1). In addition, `variance_based_decomp` and `sample_refinement` options are also supported, and the details of these specifications are identical to those described in Polynomial chaos expansion method.

Additional specifications include the level mappings described in Uncertainty Quantification Methods and the `sample_type`, `samples`, `seed`, `fixed_seed`, and `rng` specifications described in Nondeterministic sampling method. These latter sampling specifications refer to sampling on the interpolant for the purposes of generating approximate statistics, which should not be confused with collocation evaluations used for forming the interpolant. Table 5.34 provides details of the stochastic collocation specifications beyond those of Table 5.28.

### 5.7.2 Epistemic Uncertainty Quantification Methods

Epistemic uncertainty is also referred to as subjective uncertainty, reducible uncertainty, model form uncertainty, or uncertainty due to lack of knowledge. Examples of epistemic uncertainty are little or no experimental data...
### 5.7 Uncertainty Quantification Methods

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
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</tr>
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<td>askey</td>
<td>wiener</td>
<td>none</td>
<td>Optional</td>
</tr>
<tr>
<td>Automated polynomial order refinement</td>
<td>p_refinement</td>
<td>uniform</td>
<td>adaptive</td>
<td>Optional group</td>
</tr>
<tr>
<td>Adaptive refinement control</td>
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<td>generalized</td>
<td>none</td>
<td>Optional group</td>
</tr>
<tr>
<td>Quadrature order for collocation points</td>
<td>quadrature_-order</td>
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<td>Required (1 of 2 selections)</td>
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</tr>
<tr>
<td>Sparse grid level for collocation points</td>
<td>sparse_-grid_level</td>
<td>integer</td>
<td>Required (1 of 2 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Sparse grid dimension preference</td>
<td>dimension_-preference</td>
<td>list of reals</td>
<td>Optional</td>
<td>isotropic sparse grid</td>
</tr>
<tr>
<td>Integration rule point nesting</td>
<td>nested</td>
<td>non_nested</td>
<td>None</td>
<td>Optional</td>
</tr>
<tr>
<td>Sampling type</td>
<td>sample_type</td>
<td>random</td>
<td>lhs</td>
<td>Optional group</td>
</tr>
<tr>
<td>Number of samples on interpolant for generating statistics</td>
<td>samples</td>
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<td>Optional</td>
<td>0 (will result in error if sampling-based statistics are requested)</td>
</tr>
<tr>
<td>Random seed</td>
<td>seed</td>
<td>integer</td>
<td>Optional</td>
<td>randomly generated seed</td>
</tr>
<tr>
<td>Fixed seed flag</td>
<td>fixed_seed</td>
<td>none</td>
<td>Optional</td>
<td>seed not fixed: sampling patterns are variable among multiple SC runs</td>
</tr>
<tr>
<td>Random number generator</td>
<td>rng</td>
<td>mt19937</td>
<td>rnum2</td>
<td>Optional</td>
</tr>
<tr>
<td>Importance sampling refinement</td>
<td>sample_-refinement</td>
<td>import</td>
<td>adapt_import</td>
<td>Optional group</td>
</tr>
<tr>
<td>Variance-based decomposition (VBD)</td>
<td>variance_-based_decomp</td>
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<td>Optional group</td>
<td>VBD indices not computed/printed</td>
</tr>
<tr>
<td>Restriction of VBD indices to main/total</td>
<td>univariate_-effects</td>
<td>none</td>
<td>Optional</td>
<td>Unrestricted: interaction effects included</td>
</tr>
<tr>
<td>VBD tolerance for omitting small indices</td>
<td>drop_-tolerance</td>
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<td>Optional</td>
<td>All VBD indices displayed</td>
</tr>
<tr>
<td>All variables flag</td>
<td>all_-</td>
<td>none</td>
<td>Optional</td>
<td>Expansion only</td>
</tr>
</tbody>
</table>
for an unknown physical parameter, or the existence of complex physics or behavior that is not included in the simulation model of a system. Epistemic uncertainty can be modeled probabilistically but is often modeled using non-probabilistic approaches such as interval propagation, evidence theory, possibility theory, information gap theory, etc. In DAKOTA, epistemic uncertainty analysis is performed using interval analysis or Dempster-Shafer theory of evidence. Epistemic (or mixed aleatory-epistemic) uncertainty may also be propagated through the use of the Nondeterministic sampling method, although in this case, the output statistics are limited to response intervals (any epistemic component suppresses all probabilistic results). Mixed uncertainty can also be addressed through use of nested UQ (refer to the Users Manual [Adams et al., 2010] for NestedModel discussion and examples); in this case, epistemic and aleatory analyses can be segregated and intervals on probabilistic results can be reported. A subtle distinction exists between sampling for epistemic intervals and the lhs option of global_interval_est: the former allows mixed aleatory-epistemic uncertainty within a single level, whereas the latter supports only epistemic variables and relies on nesting to address mixed uncertainty. In each of these cases, the Interval Uncertain Variable specification is used to describe the epistemic uncertainty using either simple intervals or basic probability assignments. Note that for mixed UQ problems with both aleatory and epistemic variables, if the user defines the epistemic variables as intervals and aleatory variables as probability distribution types, the method sampling (in a simple, single-level study) will result in intervals only on the output. Although the aleatory variables will be sampled according to their distributions, the output will only be reported as an interval given the presence of interval variables. There is also the option to perform nested sampling, where one separates the epistemic and aleatory uncertain variables, samples over epistemic variables in the outer loop and then samples the aleatory variables in the inner loop, resulting in intervals on statistics. The calculation of intervals on statistics can also be performed by using nested approaches with interval estimation or evidence methods in the outer loop and aleatory UQ methods on the inner loop such as stochastic expansion or reliability methods. More detail about these "intervals on statistics" approaches can be found in [Eldred and Swiler, 2009] and [Eldred et al., 2011].

**Local Interval Estimation**  
In interval analysis, one assumes that nothing is known about an epistemic uncertain variable except that its value lies somewhere within an interval. In this situation, it is NOT assumed that the value has a uniform probability of occuring within the interval. Instead, the interpretation is that any value within the interval is a possible value or a potential realization of that variable. In interval analysis, the uncertainty quantification problem is one of determining the resulting bounds on the output (defining the output interval) given interval bounds on the inputs. Again, any output response that falls within the output interval is a possible output with no frequency information assigned to it.

We have the capability to perform interval analysis using either local methods (local_interval_est) or global methods (global_interval_est). If the problem is amenable to local optimization methods (e.g. can provide derivatives or use finite difference method to calculate derivatives), then one can use local methods to calculate these bounds. local_interval_est allows the user to specify either sqp which is sequential quadratic programming, or nip which is a nonlinear interior point method. Table 5.35 provides the specification for the local interval method.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
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<tr>
<td>Estimation method</td>
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<td>nip</td>
<td>none</td>
<td>Required group</td>
</tr>
</tbody>
</table>

Table 5.35: Specification detail for local interval estimation used in epistemic uncertainty

**Global Interval Estimation**  
As mentioned above, when performing interval analysis, one assumes that nothing is known about an epistemic uncertain variable except that its value lies somewhere within an interval. The goal
of uncertainty quantification in this context is to determine the resulting bounds on the output (defining the output interval) given interval bounds on the inputs.

In the global approach, one uses either a global optimization method or a sampling method to assess the bounds. `global_interval_est` allows the user to specify either `lhs`, which performs Latin Hypercube Sampling and takes the minimum and maximum of the samples as the bounds (no optimization is performed) or `ego`. In the case of `ego`, the efficient global optimization (EGO) method is used to calculate bounds (see the EGO method on this page for more explanation). When using `lhs` or `ego`, one can specify a seed for the number of LHS samples, the random number generator, and the number of samples. Table 5.36 provides the specification for the global interval methods.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
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<td>Required group</td>
</tr>
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<td>Random seed generator</td>
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<td><code>mt19937</code></td>
<td><code>rnum2</code></td>
<td>Optional</td>
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<td>Random seed</td>
<td><code>seed</code></td>
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<td>integer</td>
<td>Optional</td>
</tr>
<tr>
<td>Number of samples</td>
<td><code>samples</code></td>
<td></td>
<td>integer</td>
<td>Optional</td>
</tr>
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</table>

Table 5.36: Specification detail for global interval estimation used in epistemic uncertainty

### Local Evidence Theory (Dempster-Shafer) Methods

The above section discussed a pure interval approach. This section discusses Dempster-Shafer evidence theory. In this approach, one does not assign a probability distribution to each uncertain input variable. Rather, one divides each uncertain input variable into one or more intervals. The input parameters are only known to occur within intervals: nothing more is assumed. Each interval is defined by its upper and lower bounds, and a Basic Probability Assignment (BPA) associated with that interval. The BPA represents a probability of that uncertain variable being located within that interval. The intervals and BPAs are used to construct uncertainty measures on the outputs called "belief" and "plausibility." Belief represents the smallest possible probability that is consistent with the evidence, while plausibility represents the largest possible probability that is consistent with the evidence. For more information about the Dempster-Shafer theory of evidence, see Oberkampf and Helton, 2003 and Helton and Oberkampf, 2004.

Similar to the interval approaches, one may use global or local methods to determine plausibility and belief measures for the outputs. Note that to calculate the plausibility and belief cumulative distribution functions, one has to look at all combinations of intervals for the uncertain variables. Within each interval cell combination, the minimum and maximum value of the objective function determine the belief and plausibility, respectively. In terms of implementation, global methods use LHS sampling or global optimization to calculate the minimum and maximum values of the objective function within each interval cell, while local methods use gradient-based optimization methods to calculate these minima and maxima.

Finally, note that the nondeterministic general settings apply to the interval and evidence methods, but one needs to be careful about the interpretation and translate probabilistic measures to epistemic ones. For example, if the user specifies distribution of type complementary, a complementary plausibility and belief function will be generated for the evidence methods (as opposed to a complementary distribution function in the sampling case). If the user specifies a set of responses levels, both the belief and plausibility will be calculated for each response.
level. Likewise, if the user specifies a probability level, the probability level will be interpreted both as a belief and plausibility, and response levels corresponding to the belief and plausibility levels will be calculated. Finally, if generalized reliability levels are specified, either as inputs (gen_reliability_levels) or outputs (response_levels with compute gen_reliabilities), then these are directly converted to/from probability levels and the same probability-based mappings described above are performed.

Table 5.37 provides the specification for the local_evidence method. Note that two local optimization methods are available: sqp (sequential quadratic programming or nip (nonlinear interior point method).

<table>
<thead>
<tr>
<th>Description</th>
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<td>Estimation method</td>
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<td>nip</td>
<td>none</td>
<td>Required group</td>
</tr>
</tbody>
</table>

Table 5.37: Specification detail for local evidence theory method for epistemic uncertainty

Global Evidence Theory (Dempster-Shafer) Methods Evidence theory has been explained above in the Local Evidence Theory section. The basic idea is that one specifies an "evidence structure" on uncertain inputs and propagates that to obtain belief and plausibility functions on the response functions. The inputs are defined by sets of intervals and Basic Probability Assignments (BPAs). Evidence propagation is computationally expensive, since the minimum and maximum function value must be calculated for each "interval cell combination." These bounds are aggregated into belief and plausibility.

Table 5.38 provides the specification for the global_evidence method. global_evidence allows the user to specify either lhs or ego. lhs performs Latin Hypercube Sampling and takes the minimum and maximum of the samples as the bounds per "interval cell combination." In the case of ego, the efficient global optimization (EGO) method is used to calculate bounds (see the EGO method on this page for more explanation). When using lhs or ego, one can specify a seed for the number of LHS samples, the random number generator, and the number of samples.

Note that to calculate the plausibility and belief cumulative distribution functions, one has to look at all combinations of intervals for the uncertain variables. In terms of implementation, if one is using LHS sampling as outlined above, this method creates a large sample over the response surface, then examines each cell to determine the minimum and maximum sample values within each cell. To do this, one needs to set the number of samples relatively high: the default is 10,000 and we recommend at least that number. If the model you are running is a simulation that is computationally quite expensive, we recommend that you set up a surrogate model within the DAKOTA input file so that global_evidence performs its sampling and calculations on the surrogate and not on the original model. If one uses optimization methods instead to find the minimum and maximum sample values within each cell, this can also be computationally expensive.

5.8 Solution Verification Methods

Solution verification procedures estimate the order of convergence of the simulation response data during the course of a refinement study. This branch of methods is new and currently only contains one algorithm: Richardson extrapolation.
5.8 Solution Verification Methods

<table>
<thead>
<tr>
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</thead>
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<td>Estimation method</td>
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<td>ego</td>
<td>none</td>
<td>Required group</td>
</tr>
<tr>
<td>Random seed generator</td>
<td>rng</td>
<td>mt19937 or rnum2</td>
<td>Optional</td>
<td>Mersenne twister (mt19937)</td>
</tr>
<tr>
<td>Random seed</td>
<td>seed</td>
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<td>Optional</td>
<td>randomly generated seed</td>
</tr>
<tr>
<td>Number of samples</td>
<td>samples</td>
<td>integer</td>
<td>Optional</td>
<td>10,000 for LHS, approximately numVars^2 for EGO</td>
</tr>
</tbody>
</table>

Table 5.38: Specification detail for global evidence theory method for epistemic uncertainty

5.8.1 Richardson Extrapolation

This method utilizes state variables as active variables (continuous state only at this time; discrete state to follow later) for parameterizing the refinement of the discretizations, and employs the max_iterations and convergence_tolerance method independent controls to manage the iterative procedures. The refinement path is determined from the initial_state of the continuous_state variables specification (refer to State Variables) in combination with the refinement_rate, where each of the state variables is treated as an independent refinement factor and each of the initial state values is repeatedly divided by the refinement rate value to define new discretization states. Three algorithm options are currently provided:

1. The estimate_order option is the simplest option. For each of the refinement factors, it evaluates three points along the refinement path and uses these results to perform an estimate of the convergence order for each response function.

1. The converge_order option is initialized using the estimate_order approach, and additional refinements are performed along the refinement path until the convergence order estimates converge (two-norm of the change in response orders is less than the convergence tolerance).

1. The converge_qoi option is similar to the converge_order option, except that the convergence criterion is that the two-norm of the response discretization errors (computed from extrapolation) must be less than the convergence tolerance.

In each of these cases, convergence order for a response quantity of interest (QOI) is estimated from

\[ p = \ln\left(\frac{QOI_3 - QOI_2}{QOI_2 - QOI_1}\right)/\ln(r) \]

where \( r \) is the uniform refinement rate specified by refinement_rate. Table 5.39 provides the specification details for the richardson_extrap method.
### Method Commands

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</table>

Table 5.39: Specification detail for Richardson extrapolation method for solution verification

### 5.9 Design of Computer Experiments Methods

Design and Analysis of Computer Experiments (DACE) methods compute response data sets at a selection of points in the parameter space. Three libraries are provided for performing these studies: DDACE, FSUDace, and PSUADE. The design of experiments methods do not currently make use of any of the method independent controls.

#### 5.9.1 DDACE

The Distributed Design and Analysis of Computer Experiments (DDACE) library provides the following DACE techniques: grid sampling (grid), pure random sampling (random), orthogonal array sampling (oas), latin hypercube sampling (lhs), orthogonal array latin hypercube sampling (oa_lhs), Box-Behnken (box_benhken), and central composite design (central_composite). It is worth noting that there is some overlap in sampling techniques with those available from the nondeterministic branch. The current distinction is that the nondeterministic branch methods are designed to sample within a variety of probability distributions for uncertain variables, whereas the design of experiments methods treat all variables as having uniform distributions. As such, the design of experiments methods are well-suited for performing parametric studies and for generating data sets used in building global approximations (see Global approximations), but are not currently suited for assessing the effect of uncertainties. If a design of experiments over both design/state variables (treated as uniform) and uncertain variables (with probability distributions) is desired, then sampling can support this with its all_variables option (see Nondeterministic sampling method). DAKOTA provides access to the DDACE library through the DDACEDesignCompExp class.

In terms of method dependent controls, the specification structure is straightforward. First, there is a set of design of experiments algorithm selections separated by logical OR’s (grid or random or oas or lhs or oa_lhs or box_benhken or central_composite). Second, there are optional specifications for the random seed to use in generating the sample set (seed), for fixing the seed (fixed_seed) among multiple sample sets (see Nondeterministic sampling method for discussion), for the number of samples to perform (samples), and for the number of symbols to use (symbols). The seed control is used to make sample sets repeatable, and the symbols control is related to the number of replications in the sample set (a larger number of symbols equates to more stratification and fewer replications). The main_effects control prints Analysis-of-Variance main effects results (e.g. ANOVA tables with p-values per variable). The main_effects control is only operational with the orthogonal arrays or Latin Hypercube designs, not for Box Behnken or Central Composite designs. The quality_metrics control is available for the DDACE library. This control turns on calculation of volumetric quality measures which measure the uniformity of the point samples. More details on the quality measures are...
given under the description of the FSU sampling methods. The `variance_based_decomp` control is also available. This control enables the calculation of sensitivity indices which indicate how important the uncertainty in each input variable is in contributing to the output variance. More details on variance based decomposition are given in Nondeterministic sampling method. Design of experiments specification detail is given in Table 5.40.

<table>
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<td>Optional</td>
<td>randomly generated seed</td>
</tr>
<tr>
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<td>fixed_seed</td>
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<td>Optional</td>
<td>seed not fixed: sampling patterns are variable among multiple DACE runs</td>
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<td>minimum required</td>
</tr>
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<td>Number of symbols</td>
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<td>Optional</td>
<td>default for sampling algorithm</td>
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<td>Optional</td>
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<td>Quality metrics</td>
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<td>variance_-based_decomp</td>
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<td>No variance_-based_decomp</td>
</tr>
</tbody>
</table>

Table 5.40: Specification detail for design of experiments methods

5.9.2 FSUDace

The Florida State University Design and Analysis of Computer Experiments (FSUDace) library provides the following DACE techniques: quasi-Monte Carlo sampling (`fsu_quasi_mc`) based on the Halton sequence (`halton`) or the Hammersley sequence (`hammersley`), and Centroidal Voronoi Tessellation (`fsu_cvt`). All three methods generate sets of uniform random variables on the interval [0,1]. If the user specifies lower and upper bounds for a variable, the [0,1] samples are mapped to the [lower, upper] interval. The quasi-Monte Carlo and CVT methods are designed with the goal of low discrepancy. Discrepancy refers to the nonuniformity of the sample points within the hypercube. Discrepancy is defined as the difference between the actual number and the expected number of points one would expect in a particular set B (such as a hyper-rectangle within the unit hypercube), maximized over all such sets. Low discrepancy sequences tend to cover the unit hypercube reasonably uniformly. Quasi-Monte Carlo methods produce low discrepancy sequences, especially if one is interested in the uniformity of projections of the point sets onto lower dimensional faces of the hypercube (usually 1-D: how well do the marginal distributions approximate a uniform?) CVT does very well volumetrically: it spaces the points...
fairly equally throughout the space, so that the points cover the region and are isotropically distributed with no directional bias in the point placement. There are various measures of volumetric uniformity which take into account the distances between pairs of points, regularity measures, etc. Note that CVT does not produce low-discrepancy sequences in lower dimensions, however: the lower-dimension (such as 1-D) projections of CVT can have high discrepancy.

The quasi-Monte Carlo sequences of Halton and Hammersley are deterministic sequences determined by a set of prime bases. Generally, we recommend that the user leave the default setting for the bases, which are the lowest primes. Thus, if one wants to generate a sample set for 3 random variables, the default bases used are 2, 3, and 5 in the Halton sequence. To give an example of how these sequences look, the Halton sequence in base 2 starts with points 0.5, 0.25, 0.75, 0.125, 0.625, etc. The first few points in a Halton base 3 sequence are 0.33333, 0.66667, 0.11111, 0.44444, 0.77777, etc. Notice that the Halton sequence tends to alternate back and forth, generating a point closer to zero then a point closer to one. An individual sequence is based on a radix inverse function defined on a prime base. The prime base determines how quickly the \([0,1]\) interval is filled in. Generally, the lowest primes are recommended.

The Hammersley sequence is the same as the Halton sequence, except the values for the first random variable are equal to \(1/N\), where \(N\) is the number of samples. Thus, if one wants to generate a sample set of 100 samples for 3 random variables, the first random variable has values 1/100, 2/100, 3/100, etc. and the second and third variables are generated according to a Halton sequence with bases 2 and 3, respectively. For more information about these sequences, see [Halton, 1960, Halton and Smith, 1964, and Kocis and Whiten, 1997].

The specification for specifying quasi-Monte Carlo (fsu_quasi_mc) is given below in Table 5.41. The user must specify if the sequence is (halton) or (hammersley). The user must also specify the number of samples to generate for each variable (samples). Then, there are three optional lists the user may specify. The first list determines where in the sequence the user wants to start. For example, for the Halton sequence in base 2, if the user specifies sequence_start = 2, the sequence would not include 0.5 and 0.25, but instead would start at 0.75. The default sequence_start is a vector with 0 for each variable, specifying that each sequence start with the first term. The sequence_leap control is similar but controls the "leaping" of terms in the sequence. The default is 1 for each variable, meaning that each term in the sequence be returned. If the user specifies a sequence_leap of 2 for a variable, the points returned would be every other term from the QMC sequence. The advantage to using a leap value greater than one is mainly for high-dimensional sets of random deviates. In this case, setting a leap value to the next prime number larger than the largest prime base can help maintain uniformity when generating sample sets for high dimensions. For more information about the efficacy of leaped Halton sequences, see [Robinson and Atcitty, 1999]. The final specification for the QMC sequences is the prime base. It is recommended that the user not specify this and use the default values. For the Halton sequence, the default bases are primes in increasing order, starting with 2, 3, 5, etc. For the Hammersley sequence, the user specifies (s-1) primes if one is generating an s-dimensional set of random variables.

The fixed_sequence control is similar to fixed_seed for other sampling methods. If fixed_sequence is specified, the user will get the same sequence (meaning the same set of samples) for subsequent calls of the QMC sampling method (for example, this might be used in a surrogate based optimization method or a parameter study where one wants to fix the uncertain variables). The latinize command takes the QMC sequence and "latinizes" it, meaning that each original sample is moved so that it falls into one strata or bin in each dimension as in Latin Hypercube sampling. The default setting is NOT to latinize a QMC sample. However, one may be interested in doing this in situations where one wants better discrepancy of the 1-dimensional projections (the marginal distributions). The variance_based_decomp control is also available. This control enables the calculation of sensitivity indices which indicate how important the uncertainty in each input variable is in contributing to the output variance. More details on variance based decomposition are given in Nondeterministic sampling method.

Finally, quality_metrics calculates four quality metrics relating to the volumetric spacing of the samples.
The four quality metrics measure different aspects relating to the uniformity of point samples in hypercubes. Desirable properties of such point samples are: are the points equally spaced, do the points cover the region, and are they isotropically distributed, with no directional bias in the spacing. The four quality metrics we report are \(h\), \(\chi\), \(\tau\), and \(d\). \(h\) is the point distribution norm, which is a measure of uniformity of the point distribution. \(\chi\) is a regularity measure, and provides a measure of local uniformity of a set of points. \(\tau\) is the second moment trace measure, and \(d\) is the second moment determinant measure. All of these values are scaled so that smaller is better (the smaller the metric, the better the uniformity of the point distribution). Complete explanation of these measures can be found in [Gunzburger and Burkardt, 2004].

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<td>Optional</td>
<td>(0) for standalone sampling, (minimum required) for surrogates</td>
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</tr>
<tr>
<td>Sequence leaping indices</td>
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<td>Vector of ones</td>
</tr>
<tr>
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<td>Vector of the first s primes for s-dimensions in Halton, First (s-1) primes for Hammersley</td>
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<td>sequence not fixed: sampling patterns are variable among multiple QMC runs</td>
</tr>
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<td>Optional</td>
<td>No variance_based_decomp</td>
</tr>
<tr>
<td>Quality metrics</td>
<td>quality_metrics</td>
<td>none</td>
<td>Optional</td>
<td>No quality_metrics</td>
</tr>
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</table>

Table 5.41: Specification detail for FSU Quasi-Monte Carlo sequences

The FSU CVT method (fsu_cvt) produces a set of sample points that are (approximately) a Centroidal Voronoi Tessellation. The primary feature of such a set of points is that they have good volumetric spacing; the points tend to arrange themselves in a pattern of cells that are roughly the same shape. To produce this set of points, an almost arbitrary set of initial points is chosen, and then an internal set of iterations is carried out. These iterations repeatedly replace the current set of sample points by an estimate of the centroids of the corresponding Voronoi
subregions. [Du, Faber, and Gunzburger, 1999].

The user may generally ignore the details of this internal iteration. If control is desired, however, there are a few variables with which the user can influence the iteration. The user may specify max_iterations, the number of iterations carried out; num_trials, the number of secondary sample points generated to adjust the location of the primary sample points; and trial_type, which controls how these secondary sample points are generated. In general, the variable with the most influence on the quality of the final sample set is num_trials, which determines how well the Voronoï subregions are sampled. Generally, num_trials should be "large", certainly much bigger than the number of sample points being requested; a reasonable value might be 10,000, but values of 100,000 or 1 million are not unusual.

CVT has a seed specification similar to that in DDACE: there are optional specifications for the random seed to use in generating the sample set (seed), for fixing the seed (fixed_seed) among multiple sample sets (see Nondeterministic sampling method for discussion), and for the number of samples to perform (samples). The seed control is used to make sample sets repeatable. Finally, the user has the option to specify the method by which the trials are created to adjust the centroids. The trial_type can be one of three types: random, where points are generated randomly; halton, where points are generated according to the Halton sequence; and grid, where points are placed on a regular grid over the hyperspace.

Finally, latinization is available for CVT as with QMC. The latinize control takes the CVT sequence and "latinizes" it, meaning that each original sample is moved so that it falls into one strata or bin in each dimension as in Latin Hypercube sampling. The default setting is NOT to latinize a CVT sample. However, one may be interested in doing this in situations where one wants better discrepancy of the 1-dimensional projections (the marginal distributions). The variance_based_decomp control is also available. This control enables the calculation of sensitivity indices which indicate how important the uncertainty in each input variable is in contributing to the output variance. More details on variance based decomposition are given in Nondeterministic sampling method. The quality_metrics control is available for CVT as with QMC. This command turns on calculation of volumetric quality measures which measure the "goodness" of the uniformity of the point samples. More details on the quality measures are given under the description of the QMC methods.

The specification detail for the FSU CVT method is given in Table 5.42.

5.9.3 PSUADE

The Problem Solving Environment for Uncertainty Analysis and Design Exploration (PSUADE) is a Lawrence Livermore National Laboratory tool for metamodeling, sensitivity analysis, uncertainty quantification, and optimization. Its features include non-intrusive and parallel function evaluations, sampling and analysis methods, an integrated design and analysis framework, global optimization, numerical integration, response surfaces (MARS and higher order regressions), graphical output with Pgplo\ or Matlab, and fault tolerance [C.H. Tong, 2005].

The Morris One-At-A-Time (MOAT) method, originally proposed by Morris [M.D. Morris, 1991], is a screening method, designed to explore a computational model to distinguish between input variables that have negligible, linear and additive, or nonlinear or interaction effects on the output. The computer experiments performed consist of individually randomized designs which vary one input factor at a time to create a sample of its elementary effects. The PSUADE implementation of MOAT is selected with method keyword psuade_moat. The number of samples (samples) must be a positive integer multiple of (number of continuous design variable + 1) and will be automatically adjusted if misspecified. The number of partitions (partitions) applies to each variable being studied and must be odd (the number of MOAT levels per variable is partitions + 1). This will also be adjusted at runtime as necessary. For information on practical use of this method, see [Saltelli, et al., 2004]. The specification detail for the PSUADE MOAT method is given in Table 5.43.
### 5.9 Design of Computer Experiments Methods

<table>
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Table 5.42: Specification detail for FSU Centroidal Voronoi Tesselation sampling

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<td>randomly generated seed</td>
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Table 5.43: Specification detail for PSUADE methods
5.10 Parameter Study Methods

DAKOTA’s parameter study methods compute response data sets at a selection of points in the parameter space. These points may be specified as a vector, a list, a set of centered vectors, or a multi-dimensional grid. Capability overviews and examples of the different types of parameter studies are provided in the Users Manual [Adams et al., 2010]. DAKOTA implements all of the parameter study methods within the ParamStudy class.

With the exception of output verbosity (a setting of silent will suppress some parameter study diagnostic output), DAKOTA’s parameter study methods do not make use of the method independent controls. Therefore, the parameter study documentation which follows is limited to the method dependent controls for the vector, list, centered, and multidimensional parameter study methods.

5.10.1 Vector parameter study

DAKOTA’s vector parameter study computes response data sets at selected intervals along a vector in parameter space. It is often used for single-coordinate parameter studies (to study the effect of a single variable on a response set), but it can be used more generally for multiple coordinate vector studies (to investigate the response variations along some n-dimensional vector such as an optimizer search direction). This study is selected using the vector_parameter_study specification followed by either a final_point or a step_vector specification.

The vector for the study can be defined in several ways (refer to dakota.input.summary). First, a final_point specification, when combined with the initial values from the variables specification (in Variables Commands, see initial_point and initial_state for design and state variables as well as inferred initial values for uncertain variables), uniquely defines an n-dimensional vector’s direction and magnitude through its start and end points. The values included in the final_point specification are the actual variable values for discrete sets, not the underlying set index value. The intervals along this vector are then specified with a num_steps specification, for which the distance between the initial values and the final_point is broken into num_steps intervals of equal length. For continuous and discrete range variables, distance is measured in the actual values of the variables, but for discrete set variables (either integer or real sets for design, uncertain, or state types), distance is instead measured in index offsets. Since discrete sets may have nonuniform offsets in their enumerated set values but have uniform offsets in their index values, defining steps in terms of set indices allows for meaningful parameter study specifications for these variable types. This study performs function evaluations at both ends, making the total number of evaluations equal to num_steps+1. The study has stringent requirements on performing appropriate steps with any discrete range and discrete set variables. A num_steps specification must result in discrete range and set index steps that are integral: no remainder is currently permitted in the integer step calculation and no rounding to integer steps will occur. The final_point specification detail is given in Table 5.44.

The other technique for defining a vector in the study is the step_vector specification. This parameter study begins at the initial values and adds the increments specified in step_vector to obtain new simulation points. For discrete set types (design, uncertain, or state; real or integer), the steps are set index offsets, not steps between the actual set values. This increment process is performed num_steps times, and since the initial values are included, the total number of simulations is again equal to num_steps+1. The step_vector specification detail is given in Table 5.45.
5.10 Parameter Study Methods

<table>
<thead>
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Table 5.44: final_point specification detail for the vector parameter study

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</table>

Table 5.45: step_vector specification detail for the vector parameter study

5.10.2 List parameter study

DAKOTA’s list parameter study allows for evaluations at user selected points of interest which need not follow any particular structure. This study is selected using the list_parameter_study method specification followed by a list_of_points specification.

The number of real values in the list_of_points specification must be a multiple of the total number of variables (including continuous and discrete types) contained in the variables specification. This parameter study simply performs simulations for the first parameter set (the first \( n \) entries in the list), followed by the next parameter set (the next \( n \) entries), and so on, until the list of points has been exhausted. Since the initial values from the variables specification will not be used, they need not be specified. For discrete set types, the actual values should be specified, not the set indices, although the values will be validated for membership within the set value specifications. The list parameter study specification detail is given in Table 5.46.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>List parameter study</td>
<td>list_parameter_study</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>List of points to evaluate</td>
<td>list_of_points</td>
<td>list of reals (actual values; no set indices)</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 5.46: Specification detail for the list parameter study
5.10.3 Centered parameter study

DAKOTA’s centered parameter study computes response data sets along multiple coordinate-based vectors, one per parameter, centered about the initial values from the variables specification. This is useful for investigation of function contours with respect to each parameter individually in the vicinity of a specific point (e.g., post-optimality analysis for verification of a minimum), thereby avoiding the cost associated with a multidimensional grid. It is selected using the `centered_parameter_study` method specification followed by `step_vector` and `steps_per_variable` specifications. The `step_vector` specification provides the size of the increments for each variable (employed sequentially, not all at once as for Vector parameter study) in either actual values (continuous and discrete range) or index offsets (discrete set). The `steps_per_variable` specification provides the number of increments per variable (again, employed sequentially) in each of the plus and minus directions. The centered parameter study specification detail is given in Table 5.47.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centered parameter study</td>
<td>centered_parameter_study</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Step vector</td>
<td>step_vector</td>
<td>list of reals (index offset components are cast to integers)</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of steps per variable</td>
<td>steps_per_variable</td>
<td>list of integers</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 5.47: Specification detail for the centered parameter study

5.10.4 Multidimensional parameter study

DAKOTA’s multidimensional parameter study computes response data sets for an n-dimensional grid of points. Each continuous and discrete range variable is partitioned into equally spaced intervals between its upper and lower bounds, each discrete set variable is partitioned into equally spaced index intervals, and each combination of the values defined by the boundaries of these partitions is evaluated.

This study is selected using the `multidim_parameter_study` method specification followed by a `partitions` specification, where the `partitions` list specifies the number of partitions for each variable. The number of entries in the partitions list must be equal to the total number of variables contained in the variables specification. As for the vector and centered studies, remainders within the integer division of the step calculations are not permitted for discrete range or set types and therefore no integer rounding occurs, so the partitions specification must be carefully selected in the presence of these types. Since the initial values from the variables specification will not be used, they need not be specified. The multidimensional parameter study specification detail is given in Table 5.48.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multidimensional parameter study</td>
<td>multidim_parameter_study</td>
<td>none</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Partitions per variable</td>
<td>partitions</td>
<td>list of integers</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 5.48: Specification detail for the multidimensional parameter study
Chapter 6

Model Commands

6.1 Model Description

A DAKOTA model specifies how variables are mapped into a set of responses in support of an iterative method and the model specification in an input file dictates the components to be used in constructing a particular model instance. This specification selects a Model from the model hierarchy, which includes SingleModel, DataFitSurrModel, HierarchSurrModel, and NestedModel derived classes. Depending on the type of derived model, different sub-specifications are needed to construct different components of the model.

Several examples follow. The first example shows a minimal specification for a single model, which is the default model when no models are specified by the user.

model,
single

This example does not provide any pointer strings and therefore relies on the default behavior of constructing the model with the last variables, interface, and responses specifications parsed.

The next example displays a surrogate model specification which selects a quadratic polynomial from among the global approximation methods. It uses a pointer to a design of experiments method for generating the data needed for building the global approximation, reuses any old data available for the current approximation region, and employs the first-order multiplicative approach to correcting the approximation at the center of the current approximation region.

model,
id_model = 'M1'
variables_pointer = 'V1'
responses_pointer = 'R1'
surrogate global
   polynomial quadratic
   dace_method_pointer = 'DACE'
   reuse_samples region
   correction multiplicative first_order

This example demonstrates the use of identifiers and pointers. It provides the optional model independent specifications for model identifier, variables pointer, and responses pointer (see Model Independent Controls) as well as model dependent specifications for global surrogates (see Global approximations).
Finally, an advanced nested model example would be

```plaintext
model,
  id_model = 'M1'
  variables_pointer = 'V1'
  responses_pointer = 'R1'
  nested
    optional_interface_pointer = 'OI1'
    optional_interface_responses_pointer = 'OIR1'
    sub_method_pointer = 'SM1'
    primary_variable_mapping = 'X' 'Y'
    secondary_variable_mapping = 'mean' 'mean'
    primary_response_mapping = 1. 0. 0. 0. 0. 0. 0. 0. 0.
    secondary_response_mapping = 0. 0. 0. 1. 3. 0. 0. 0. 0.
  0. 0. 0. 0. 0. 1. 3. 0.
```

This example also supplies model independent controls for model identifier, variables pointer, and responses pointer (see Model Independent Controls), and supplies model dependent controls for specifying details of the nested mapping (see Nested Model Controls).

### 6.2 Model Specification

As alluded to in the examples above, the model specification has the following structure:

```plaintext
model,
  <model independent controls>
  <model selection>
    <model dependent controls>
```

The `<model independent controls>` are those controls which are valid for all models. Referring to `dakota.input.summary`, these controls are defined externally from and prior to the model selection blocks. The model selection blocks are all required group specifications separated by logical OR’s, where the model selection must be single OR surrogate OR nested. If a surrogate model is specified, a secondary selection must be made for its type: global, multipoint, local, or hierarchical. The `<model dependent controls>` are those controls which are only meaningful for a specific model. These controls are defined within each model selection block. Defaults for model independent and model dependent controls are defined in `DataModel`. The following sections provide additional detail on the model independent controls followed by the model selections and their corresponding model dependent controls.

### 6.3 Model Independent Controls

The model independent controls include a model identifier string, pointers to variables and responses specifications, and a model type specification. The model identifier string is supplied with `id_model` and is used to provide a unique identifier string for use within method specifications (refer to `model_pointer` in Method Independent Controls).

The type of model can be single, nested, or surrogate. Each of these model specifications supports `variables_pointer` and `responses_pointer` strings for identifying the variables and responses specifications used in constructing the model (by cross-referencing with `id_variables` and `id_responses`
6.4 Single Model Controls

Strings from particular variables and responses keyword specifications. These pointers are valid for each model type since each model contains a set of variables that is mapped into a set of responses – only the specifics of the mapping differ. Additional pointers are used for each model type for constructing the components of the variable to response mapping. As a strategy specification identifies one or more methods and a method specification identifies a model, a model specification identifies variables, interface, and responses specifications. This top-down flow specifies all of the object interrelationships.

For each of these pointer specifications, if a pointer string is specified and no corresponding id string is available, DAKOTA will exit with an error message. If the pointer is optional and no pointer string is specified, then the last specification parsed will be used. It is appropriate to omit optional cross-referencing whenever the relationships are unambiguous due to the presence of only one specification.

Table 6.1 provides the specification detail for the model independent controls involving identifiers, model type controls, and pointers.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model set identifier</td>
<td>id_model</td>
<td>string</td>
<td>Optional</td>
<td>method use of last model parsed</td>
</tr>
<tr>
<td>Model type</td>
<td>single</td>
<td>surrogate</td>
<td>nested</td>
<td>none</td>
</tr>
<tr>
<td>Variables set pointer</td>
<td>variables_pointer</td>
<td>string</td>
<td>Optional</td>
<td>model use of last variables parsed</td>
</tr>
<tr>
<td>Responses set pointer</td>
<td>responses_pointer</td>
<td>string</td>
<td>Optional</td>
<td>model use of last responses parsed</td>
</tr>
</tbody>
</table>

Table 6.1: Specification detail for the model independent controls: identifiers, model types, and pointers

6.4 Single Model Controls

In the single model case, a single interface is used to map the variables into responses. The optional interface_pointer specification identifies this interface by cross-referencing with the id_interface string input from a particular interface keyword specification.

Table 6.2 provides the specification detail for single models.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interface set pointer</td>
<td>interface_pointer</td>
<td>string</td>
<td>Optional</td>
<td>model use of last interface parsed</td>
</tr>
</tbody>
</table>

Table 6.2: Specification detail for single models

6.5 Surrogate Model Controls

In the surrogate model case, the specification first allows a mixture of surrogate and actual response mappings through the use of the optional id_surrogates specification. This identifies the subset of the response functions by number that are to be approximated (the default is all functions). The valid response function identifiers
range from 1 through the total number of response functions (see Function Specification). Next, the specification selects a global, multipoint, local, or hierarchical approximation. Table 6.3 provides the specification detail for surrogate models.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surrogate response ids</td>
<td>id_surrogates</td>
<td>list of integers</td>
<td>Optional</td>
<td>All response functions are approximated</td>
</tr>
<tr>
<td>Surrogate type selection</td>
<td>global</td>
<td>multipoint</td>
<td>local</td>
<td>hierarchical</td>
</tr>
</tbody>
</table>

Table 6.3: Specification detail for the surrogate models

Each of these surrogate types provides an approximate representation of a "truth" model which is used to perform the parameter to response mappings. This approximation is built and updated using data from the truth model. This data is generated in some cases using a design of experiments iterator applied to the truth model (global approximations with a dace_method_pointer). In other cases, truth model data from a single point (local, hierarchical approximations), from a few previously evaluated points (multipoint approximations), or from the restart database (global approximations with reuse_samples) can be used. Surrogate models are used extensively in the surrogate-based optimization and least squares methods (see SurrBasedMinimizer and its derived classes and Surrogate-Based Minimization Methods), in which the goals are to reduce expense by minimizing the number of truth function evaluations and to smooth out noisy data with a global data fit. However, the use of surrogate models is not restricted to optimization techniques; uncertainty quantification and optimization under uncertainty methods are other primary users.

The following sections present the global, multipoint, local, and hierarchical specification groups in further detail.

6.5.1 Global approximations

The global surrogate model requires specification of one of the following approximation types: polynomial, gaussian_process, kriging, neural_network, mars, moving_least_squares, or radial_basis. These specifications will create a polynomial, Gaussian process, kriging interpolation (significantly updated as of DAKOTA 5.1), layered perceptron artificial neural network approximation, multivariate adaptive regression spline, moving least squares, or radial basis function approximation, respectively. All these approximations are implemented in SurfPack except the Gaussian process model, which is implemented directly in DAKOTA.

In the polynomial case, the order of the polynomial (linear, quadratic, or cubic) must be specified.

While historically the gaussian_process and kriging models were drastically different, as of DAKOTA 5.1, their features and implementations are more similar. The trend function of both the kriging and Gaussian process models is selected using the trend keyword, with options constant, linear, or reduced_quadratic (default). The kriging approximation has the additional option of (full) quadratic. The reduced_quadratic trend function includes the main effects, but not mixed/interaction terms; prior to DAKOTA 5.1, it was selected for the Gaussian process via the quadratic option.

For the gaussian_process approximation, the correlation parameters are always chosen by an internal global optimization procedure. Ill-conditioning is a common challenge in the construction of Gaussian process models.
The **gaussian_process** deals with ill-conditioning in two ways. First, when it encounters a non-invertible correlation matrix it iteratively increases the size of a "nugget," but in such cases the resulting approximation smooths rather than interpolates the data. Second, the GP has an option for **point_selection** to avoid ill-conditioning when the number of points becomes large. This option uses a greedy algorithm to choose a subset of points with which to build the Gaussian process. These points are "optimal" in terms of creating a Gaussian process with low prediction error. By default, there is no point selection. Typically, one should not need point selection in trust-region methods because a small number of points are used to develop a surrogate within each trust region. Point selection is most beneficial when constructing with a "large" number of points, typically more than one hundred, though this depends on the number of variables and spacing of the sample points.

By default, the kriging model uses a global optimization algorithm to determine the correlation parameters, however one may force a particular approach by specifying **optimization_method** followed by any of 'global', 'local', 'sampling', or 'none'. Alternately, a vector of **correlation_lengths** can be specified, bypassing their automatic determination. (A correlation parameter equals one-half divided by the square of the corresponding correlation length. This definition of the correlation lengths makes their effect on the Gaussian correlation function directly analogous to that of the standard deviation of a Gaussian (normal) distribution.) When performing optimization to determine correlation parameters, the Kriging model searches in a much smaller "feasible region" of correlation lengths. If the 'none' option is selected, and correlation_lengths are not directly specified, then the center (in log scale) of the feasible region will be selected. This center is also used as the starting point for 'local' optimization. The small feasible region typically contains small/large enough correlation lengths to include both well-conditioned correlation matrices and either the globally optimal, or at least a near-optimal, correlation matrix. Ill-conditioned correlation matrices are explicitly excluded from consideration based on (an estimate of) their condition number. The **max_trials** option limits the number of internal likelihood function evaluations used in constructing the kriging model.

Table 6.4 summarizes the global approximation selection specifications.

Table 6.5 summarizes the remaining global approximation specifications, including DACE method and sample reuse, correction, and use of gradient data.

For each of the global surrogates, **dace_method_pointer**, **reuse_samples**, **correction**, and **use_gradients** can be optionally specified. The **dace_method_pointer** specification points to a design of experiments iterator which can be used to generate truth model data for building a global data fit. The **reuse_samples** specification can be used to employ old data (either from previous function evaluations performed in the run or from function evaluations read from a restart database or text file) in the building of new global approximations. The default is no reuse of old data (since this can induce directional bias), and the settings of **all**, **region**, and **samples_file** result in reuse of all available data, reuse of all data available in the current trust region, and reuse of all data from a specified text file, respectively. The combination of new build data from **dace_method_pointer** and old build data from **reuse_samples** must be sufficient for building the global approximation. If not enough data is available, the system will abort with an error message. Both **dace_method_pointer** and **reuse_samples** are optional specifications, which gives the user maximum flexibility in using design of experiments data, restart/text file data, or both.

To assess the goodness of fit of a global surrogate, a variety of diagnostics are available for the following global approximation methods: polynomial regressions, kriging, mars, moving least squares, neural networks, and radial basis functions. The diagnostics are specified by the keyword **diagnostics**, followed by a list of strings specifying the actual metrics. The diagnostic metrics available are: sum_squared, mean_squared, root_mean_squared, max_squared, sum_scaled, max_scaled, sum_abs, mean_abs, max_abs, press, cv, and rsquared. Most of these diagnostics refer to some operation on the residuals (the difference between the surrogate model and the truth model at the data points upon which the surrogate is built). For example, sum_squared refers to the sum of the squared residuals, mean_abs refers to the mean of the absolute value of the residuals, and max_scaled refers to the maximum scaled value of the residual, where the scaled
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global approximations</td>
<td>global</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Polynomial</td>
<td>polynomial</td>
<td>linear</td>
<td>quadratic</td>
<td>cubic</td>
</tr>
<tr>
<td>Gaussian process</td>
<td>gaussian_-process</td>
<td>none</td>
<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>GP trend function</td>
<td>trend</td>
<td>constant</td>
<td>linear</td>
<td>reduced_- quadratic</td>
</tr>
<tr>
<td>GP point selection</td>
<td>point_-selection</td>
<td>none</td>
<td>Optional</td>
<td>no point selection</td>
</tr>
<tr>
<td>Kriging interpolation</td>
<td>kriging</td>
<td>none</td>
<td>Required group (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Kriging trend function</td>
<td>trend</td>
<td>constant</td>
<td>linear</td>
<td>reduced_- quadratic</td>
</tr>
<tr>
<td>Kriging correlation lengths</td>
<td>correlation_lengths</td>
<td>list of reals</td>
<td>Optional</td>
<td>internally computed correlation_lengths</td>
</tr>
<tr>
<td>Kriging optimization method</td>
<td>optimization_method</td>
<td>'global'</td>
<td>'_local'</td>
<td>'sampling'</td>
</tr>
<tr>
<td>Kriging maximum trials</td>
<td>max_trials</td>
<td>integer</td>
<td>Optional</td>
<td></td>
</tr>
<tr>
<td>Artificial neural network</td>
<td>neural_-network</td>
<td>none</td>
<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>ANN number nodes</td>
<td>nodes</td>
<td>integer</td>
<td>Optional</td>
<td></td>
</tr>
<tr>
<td>ANN range</td>
<td>range</td>
<td>real</td>
<td>Optional</td>
<td></td>
</tr>
<tr>
<td>ANN random weight</td>
<td>random_-weight</td>
<td>integer</td>
<td>Optional</td>
<td></td>
</tr>
<tr>
<td>Multivariate adaptive regression splines</td>
<td>mars</td>
<td>none</td>
<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>MARS maximum bases</td>
<td>max_bases</td>
<td>integer</td>
<td>Optional</td>
<td></td>
</tr>
<tr>
<td>MARS interpolation</td>
<td>interpolation</td>
<td>linear</td>
<td>cubic</td>
<td>Optional</td>
</tr>
<tr>
<td>Moving least squares</td>
<td>moving_-least_-squares</td>
<td>none</td>
<td>Required (1 of 7 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>MLS polynomial order</td>
<td>poly_order</td>
<td>integer</td>
<td>Optional</td>
<td></td>
</tr>
</tbody>
</table>
value is taken by dividing the residual by the true function value at that point. In addition, \texttt{press} refers to a leave-one-out cross validation metric, and \texttt{cv} refers to a general cross validation metric. \texttt{rsquared} refers to the R-squared value typically used in regression analysis (the proportion of the variability in the response that can be accounted for by the surrogate model).

The \texttt{correction} specification specifies that the approximation will be corrected to match truth data, either matching truth values in the case of \texttt{zeroth_order} matching, matching truth values and gradients in the case of \texttt{first_order} matching, or matching truth values, gradients, and Hessians in the case of \texttt{second_order} matching. For additive and multiplicative corrections, the correction is local in that the truth data is matched at a single point, typically the center of the approximation region. The additive correction adds a scalar offset (\texttt{zeroth_order}), a linear function (\texttt{first_order}), or a quadratic function (\texttt{second_order}) to the approximation to match the truth data at the point, and the multiplicative correction multiplies the approximation by a scalar (\texttt{zeroth_order}), a linear function (\texttt{first_order}), or a quadratic function (\texttt{second_order}) to match the truth data at the point. The additive \texttt{first_order} case is due to \cite{Lewis and Nash, 2000} and the multiplicative \texttt{first_order} case is commonly known as beta correction \cite{Haftka, 1991}. For the combined correction, the use of both additive and multiplicative corrections allows the satisfaction of an additional matching condition, typically the truth function values at the previous correction point (e.g., the center of the previous trust region). The combined correction is then a multipoint correction, as opposed to the local additive and multiplicative corrections. Each of these correction capabilities is described in detail in \cite{Eldred et al., 2004a}.

Finally, the \texttt{use_gradients} flag specifies a future capability for the use of gradient data in the global approximation builds. This capability is currently supported in \texttt{SurrBasedLocalMinimizer}, \texttt{Pecos::SurrogateDataPoint}, and \texttt{Approximation::build()}, but is not yet supported in any global approximation derived class redefinitions of \texttt{build()}.

### 6.5.2 Multipoint approximations

Multipoint approximations use data from previous design points to improve the accuracy of local approximations. Currently, the Two-point Adaptive Nonlinearity Approximation (TANA-3) method of \cite{Xu and Grandhi, 1998} is supported. This method requires response value and gradient information from two points, and uses a first-order Taylor series if only one point is available. The truth model to be used to generate the value/gradient data used in the approximation is identified through the required \texttt{actual_model_pointer} specification. Table 6.6 summarizes the multipoint approximation specifications.

### 6.5.3 Local approximations

Local approximations use value, gradient, and possibly Hessian data from a single point to form a series expansion for approximating data in the vicinity of this point. The currently available local approximation is the \texttt{taylor-series} selection. The order of the Taylor series may be either first-order or second-order, which is automatically determined from the gradient and Hessian specifications in the responses specification (see \texttt{Gradient Specification} and \texttt{Hessian Specification}) for the truth model.

The truth model to be used to generate the value/gradient/Hessian data used in the series expansion is identified through the required \texttt{actual_model_pointer} specification. The use of a model pointer (as opposed to an interface pointer) allows additional flexibility in defining the approximation. In particular, the derivative specification for the truth model may differ from the derivative specification for the approximation, and the truth model results being approximated may involve a model recursion (e.g., the values/derivatives from a nested model). Table 6.7 summarizes the local approximation interface specifications.
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design of experiments</td>
<td>dace_-method_-pointer</td>
<td>string</td>
<td>Optional</td>
<td>no design of experiments data</td>
</tr>
<tr>
<td>method pointer</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample reuse in global approximation</td>
<td>reuse_-samples</td>
<td>all</td>
<td>region</td>
<td>none</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>File import of samples for global approximation builds</td>
<td>samples_file</td>
<td>string</td>
<td>Optional</td>
<td>no sample import from a file</td>
</tr>
<tr>
<td>Surrogate correction approach</td>
<td>correction</td>
<td>additive or multiplicative or combined, zeroth_order or first_order or second_order</td>
<td>Optional group</td>
<td>no surrogate correction</td>
</tr>
<tr>
<td>Use of gradient data in global approximation builds</td>
<td>use_-gradients</td>
<td>none</td>
<td>Optional</td>
<td>gradient data not used in global approximation builds</td>
</tr>
<tr>
<td>Print diagnostic metrics about the surrogate goodness of fit</td>
<td>diagnostics</td>
<td>string list (as many as desired): sum_squared, mean_squared, root_mean_squared, max_squared, sum_scaled, max_scaled, sum_abs, mean_abs, max_abs, press, cv, and rsquared</td>
<td>Optional</td>
<td>No diagnostics printed</td>
</tr>
</tbody>
</table>

Table 6.5: Specification detail for global approximations: build and correction controls
### 6.6 Nested Model Controls

Instead of appealing directly to a primary interface, a nested model maps parameters to responses by executing a sub-iterator for each evaluation of the nested model; the sub-iterator in turn operates on a sub-model. The sub-iterator responses may be combined with those from an optional interface specification.

In the nested model case, a sub_method_pointer must be provided in order to specify the nested iterator, and optional_interface_pointer and optional_interface_responses_pointer provide an optional group specification for the optional interface portion of nested models (where optional_-

---

### Table 6.6: Specification detail for multipoint approximations

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multipoint approximation</td>
<td>multipoint</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Two-point adaptive nonlinear approximation</td>
<td>tana</td>
<td>none</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Pointer to the truth model specification</td>
<td>actual_model_pointer</td>
<td>string</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

---

### Table 6.7: Specification detail for local approximations

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local approximation</td>
<td>local</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Taylor series local approximation</td>
<td>taylor_series</td>
<td>none</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Pointer to the truth model specification</td>
<td>actual_model_pointer</td>
<td>string</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

---

### 6.5.4 Hierarchical approximations

Hierarchical approximations use corrected results from a low fidelity model as an approximation to the results of a high fidelity "truth" model. These approximations are also known as model hierarchy, multifidelity, variable fidelity, and variable complexity approximations. The required low_fidelity_model_pointer specification points to the low fidelity model specification. This model is used to generate low fidelity responses which are then corrected and returned to an iterator. The required high_fidelity_model_pointer specification points to the specification for the high fidelity truth model. This model is used only for verifying low fidelity results and updating low fidelity corrections. The correction specification specifies which correction technique will be applied to the low fidelity results in order to match the high fidelity results at one or more points. In the hierarchical case (as compared to the global case), the correction specification is required, since the omission of a correction technique would effectively eliminate the purpose of the high fidelity model. If it is desired to use a low fidelity model without corrections, then a hierarchical approximation is not needed and a single model should be used. Refer to Global approximations for additional information on available correction approaches. Table 6.8 summarizes the hierarchical approximation specifications.
Hierarchical approximation

**Description**

**Keyword**

**Associated Data**

**Status**

**Default**

---

Optional group (1 of 4 selections)

N/A

---

low_fidelity_model_pointer

string

Required

N/A

---

high_fidelity_model_pointer

string

Required

N/A

---

surrogate_correction

correction

additive or multiplicative or combined, zeroth_order or first_order or second_order

Required group

N/A

---

Table 6.8: Specification detail for hierarchical approximations

**interface_pointer** points to the interface specification and **optional_interface_responses_pointer** points to a responses specification describing the data to be returned by this interface). This interface is used to provide non-nested data, which is then combined with data from the nested iterator using the **primary_response_mapping** and **secondary_response_mapping** inputs (see mapping discussion below).

Table 6.9 provides the specification detail for nested model pointers.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optional interface set pointer</td>
<td>optional_interface_pointer</td>
<td>string</td>
<td>Optional</td>
<td>no optional interface</td>
</tr>
<tr>
<td>Responses pointer for nested model</td>
<td>optional_interface_responses_pointer</td>
<td>string</td>
<td>Optional</td>
<td>reuse of top-level responses specification</td>
</tr>
<tr>
<td>optional interfaces</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-method pointer for nested models</td>
<td>sub_method_pointer</td>
<td>string</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 6.9: Specification detail for nested models

Nested models may employ mappings for both the variable inputs to the sub-model and the response outputs from the sub-model. In the former case, the **primary_variable_mapping** and **secondary_variable_mapping** specifications are used to map from the active top-level variables into the sub-model variables, and in the latter case, the **primary_response_mapping** and **secondary_response_mapping** specifications are used to compute the sub-model response contributions to the top-level responses. For the variable mappings, the primary and secondary specifications provide lists of strings which are used to target specific sub-model variables and their sub-parameters, respectively. The primary strings are matched to continuous or discrete variable
labels such as ‘cdv_1’ (either user-supplied or default labels), and the secondary strings are matched to either real or integer random variable distribution parameters such as ‘mean’ or ‘num_trials’ (the form of the uncertain distribution parameter keyword that is appropriate for a single variable instance) or continuous or discrete design/state variable sub-parameters such as ‘lower_bound’ or ‘upper_bound’ (again, keyword form appropriate for a single variable instance). No coercion of types is supported, so real-valued top-level variables should map to either real-valued sub-model variables or real-valued sub-parameters and integer-valued top-level variables should map to either integer-valued sub-model variables or integer-valued sub-parameters. As long as these real versus integer constraints are satisfied, mappings are free to cross variable types (design, aleatory uncertain, epistemic uncertain, state) and domain types (continuous, discrete). Both primary_variable_mapping and secondary_variable_mapping specifications are optional, which is designed to support the following three possibilities:

1. If both primary and secondary variable mappings are specified, then an active top-level variable value will be inserted into the identified sub-parameter (the secondary mapping) for the identified sub-model variable (the primary mapping).

2. If a primary mapping is specified but a secondary mapping is not, then an active top-level variable value will be inserted into the identified sub-model variable value (the primary mapping).

3. If a primary mapping is not specified (corresponding secondary mappings, if specified, are ignored), then an active top-level variable value will be inserted into a corresponding sub-model variable, based on matching of variable types (e.g., top-level and sub-model variable specifications both allocate a set of ‘continuous_design’ variables which are active at the top level). Multiple sub-model variable types may be updated in this manner, provided that they are all active in the top-level variables. Since there is a direct variable correspondence for these default insertions, sub-model bounds and labels are also updated from the top-level bounds and labels in order to eliminate the need for redundant input file specifications. Thus, it is typical for the sub-model variables specification to only contain the minimal required information, such as the number of variables, for these insertion targets. The sub-model must allocate enough space for each of the types that will accept default insertions, and the leading set of matching sub-model variables are updated (i.e., the sub-model may allocate more than needed and the trailing set will be unmodified).

These different variable mapping possibilities may be used in any combination by employing empty strings ("") for particular omitted mappings (the number of strings in user-supplied primary and secondary variable mapping specifications must equal the total number of active top-level variables, including both continuous and discrete types). The ordering of the active variables is the same as shown in dakota.input.summary and as presented in Variables Commands.

If inactive variables are present at the outer level, then the default type 3 mapping is used for these variables; that is, outer loop inactive variables are inserted into inner loop variables (active or inactive) based on matching of variable types, top-level bounds and labels are also propagated, the inner loop must allocate sufficient space to receive the outer loop values, and the leading subset within this inner loop allocation is updated. This capability is important for allowing nesting beyond two levels, since an active variable at the outer-most loop may become inactive at the next lower level, but still need to be further propagated down the recursion.

For the response mappings, the primary and secondary specifications provide real-valued multipliers to be applied to sub-iterator response results. The idea is that the responses from the inner loop are mapped to the outer loop. For example, if the nested model is an uncertainty quantification, aleatory statistics from the inner loop such as the mean inner loop response are mapped to the outer level, where they are treated epistemically, so that intervals on the mean (for example) are calculated. The response mapping defines a vector which multiplies the values from the inner loop to the outer loop. Each row of the mapping corresponds to one outer loop response, where each column of the mapping corresponds to a value from the inner loop. Depending on the number of responses
and the particular attributes calculated on the inner loop, there will be a vector of inner loop response values that need to be accounted for in the mapping. The sub-iterator response results are defined as follows for different sub-iterator types:

- **optimization**: the final objective function(s) and nonlinear constraints
- **nonlinear least squares**: the final least squares terms and nonlinear constraints
- **aleatory uncertainty quantification (UQ)**: for each response function, a mean statistic, a standard deviation statistic, and all probability/reliability/generalized reliability/response level results for any user-specified `response_levels`, `probability_levels`, `reliability_levels`, and/or `gen_reliability_levels`, in that order.
- **epistemic and mixed aleatory/epistemic UQ using interval estimation methods**: lower and upper interval bounds for each response function.
- **epistemic and mixed aleatory/epistemic UQ using evidence methods**: for each response function, lower and upper interval bounds (belief and plausibility) for all probability/reliability/generalized reliability/response level results computed from any user-specified `response_levels`, `probability_levels`, `reliability_levels`, and/or `gen_reliability_levels`, in that order.
- **parameter studies and design of experiments**: for optimization and least squares response data sets, the best solution found (lowest constraint violation if infeasible, lowest composite objective function if feasible). For generic response data sets, a best solution metric is not defined, so sub-iterator response results are not defined in this case.

The primary values map sub-iterator response results into top-level objective functions, least squares terms, or generic response functions, depending on the declared top-level response set. The secondary values map sub-iterator response results into top-level nonlinear inequality and equality constraints. Refer to `NestedModel::response_mapping()` for additional details.

An example of variable and response mappings is provided below:

```plaintext
primary_variable_mapping = '' '' 'X' 'Y'
secondary_variable_mapping = '' '' 'mean' 'mean'
primary_response_mapping = 1. 0. 0. 0. 0. 0. 0. 0. 0.
secondary_response_mapping = 0. 0. 1. 3. 0. 0. 0. 0. 0.
```

The variable mappings correspond to 4 top-level variables, the first two of which employ the default mappings from active top-level variables to sub-model variables of the same type (option 3 above) and the latter two of which are inserted into the mean distribution parameters of sub-model variables 'X' and 'Y' (option 1 above). In this particular example, there are 9 inner loop response attributes and 3 outer loop response functions (one primary response function and 2 secondary functions, such as one objective and two constraints). Each row of the response mapping is a vector which is multiplied (e.g. dot-product) against the 9 sub-iterator values to determine the outer loop function. For example, the primary response mapping only picks up the first value from the inner loop. This first value is the mean of the first response function on the inner loop. The response mappings correspond to 9 sub-iterator response functions (e.g., there are 3 response functions, each with a set of UQ final statistics: each has a mean, a standard deviation, and a level mapping in this example. If no probability or reliability levels are specified, the responses would only have a mean and standard deviation). The primary response mapping maps the first sub-iterator response function (mean) into a single objective function, least squares term, or generic response function.
(as dictated by the top-level response specification), and the secondary response mapping maps the fourth
sub-iterator response function plus 3 times the fifth sub-iterator response function (mean plus 3 standard deviations)
into one top-level nonlinear constraint and the seventh sub-iterator response function plus 3 times the eighth sub-
iterator response function (mean plus 3 standard deviations) into another top-level nonlinear constraint (these
top-level nonlinear constraints may be inequality or equality, as dictated by the top-level response specification).
Note that in many cases, each particular sub-iterator response will be mapped to a unique outer loop response (for
example, in the nested UQ case where one wants to determine an interval on each inner loop statistic). In these
cases, the response mapping will be the identity matrix. That is, the primary response mapping will have N rows
and N columns corresponding to the N sub-iterator response values, and the mapping matrix will have a value of
one along the diagonal and zeros elsewhere.

Table 6.10 provides the specification detail for the model independent controls involving nested model mappings.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary variable mappings for nested models</td>
<td>primary_variable_mapping</td>
<td>list of strings</td>
<td>Optional</td>
<td>default variable insertions based on variable type</td>
</tr>
<tr>
<td>Secondary variable mappings for nested models</td>
<td>secondary_variable_mapping</td>
<td>list of strings</td>
<td>Optional</td>
<td>primary mappings into sub-model variables are value-based</td>
</tr>
<tr>
<td>Primary response mappings for nested models</td>
<td>primary_response_mapping</td>
<td>list of reals</td>
<td>Optional</td>
<td>no sub-iterator contribution to primary functions</td>
</tr>
<tr>
<td>Secondary response mappings for nested models</td>
<td>secondary_response_mapping</td>
<td>list of reals</td>
<td>Optional</td>
<td>no sub-iterator contribution to secondary functions</td>
</tr>
</tbody>
</table>

Table 6.10: Specification detail for the model independent controls: nested model mappings
Chapter 7

Variables Commands

7.1 Variables Description

The variables section in a DAKOTA input file specifies the parameter set to be iterated by a particular method. This parameter set is made up of design, uncertain, and state variables. Design variables can be continuous or discrete and consist of those variables which an optimizer adjusts in order to locate an optimal design. Each of the design parameters can have an initial point and a descriptive tag. Continuous and discrete range types include lower and upper bounds, and discrete set types include the admissible set values.

Uncertain variables may be categorized as either aleatory or epistemic and either continuous or discrete. Continuous aleatory uncertain variables include normal, lognormal, uniform, loguniform, triangular, exponential, beta, gamma, gumbel, frechet, weibull, and histogram bin distributions. Discrete aleatory uncertain variables include poisson, binomial, negative binomial, geometric, hypergeometric, and histogram point distributions. In addition to aleatory uncertain variables defined by probability distributions, DAKOTA also supports epistemic uncertain variables that are non-probabilistic. The interval type specification is a continuous epistemic type that supports both simple bounded intervals as well as basic probability assignment (BPA) belief structures, where a BPA defines the uncertainty in a variable through providing one or more intervals in which the variable may lie along with varying levels of belief for each interval.

Each uncertain variable specification contains descriptive tags and most contain, either explicitly or implicitly, distribution lower and upper bounds. Distribution lower and upper bounds are explicit portions of the normal, lognormal, uniform, loguniform, triangular, and beta specifications, whereas they are implicitly defined for histogram bin, histogram point, and interval variables (from the extreme values within the bin/point/interval specifications) as well as for binomial (0 to num_trials) and hypergeometric (0 to min(num_drawn, num_selected)) variables. When used with design of experiments and multidimensional parameter studies, distribution bounds are also inferred for normal and lognormal (if optional bounds are unspecified) as well as for exponential, gamma, gumbel, frechet, weibull, poisson, negative binomial, and geometric (which have no bounds specifications); these bounds are \([0, \mu + 3\sigma]\) for exponential, gamma, frechet, weibull, poisson, negative binomial, geometric, and unspecified lognormal, and \([\mu - 3\sigma, \mu + 3\sigma]\) for gumbel and unspecified normal. For other types of parameter studies (vector and centered), an inferred initial starting point is needed for the uncertain variables. All uncertain variables are initialized to their means for these studies, where mean values for bounded normal and bounded lognormal may additionally be repaired to satisfy any specified distribution bounds, mean values for discrete integer range distributions are rounded down to the nearest integer, and mean values for discrete set distributions are rounded to the nearest set value.
In addition to tags and bounds specifications, normal variables include mean and standard deviation specifications, lognormal variables include lambda and zeta, mean and standard deviation, or mean and error factor specifications, triangular variables include mode specifications, exponential variables include beta specifications, beta, gamma, gumbel, frechet, and weibull variables include alpha and beta specifications, histogram bin variables include abscissa and either ordinate or count specifications, poisson variables include lambda specifications, binomial and negative binomial variables include probability per trial and number of trials specifications, geometric variables include probability per trial specifications, hypergeometric variables include the specification of the total population, selected population, and number drawn, histogram point variables include abscissa and count specifications, and interval variables include basic probability assignments per interval.

State variables can be continuous or discrete and consist of "other" variables which are to be mapped through the simulation interface. Each state variable specification can have an initial state and descriptors. Continuous and discrete range types include lower and upper bounds, and discrete set types include the admissible set values. State variables provide a convenient mechanism for parameterizing additional model inputs, such as mesh density, simulation convergence tolerances and time step controls, and can be used to enact model adaptivity in future strategy developments.

The ordering of variables is important, and a consistent ordering is employed throughout the DAKOTA software. It is the same ordering as shown in `dakota.input.summary` and as presented in the outline of this chapter. That ordering can be summarized as continuous followed by discrete integer followed by discrete real within each of the following types: design, aleatory uncertain, epistemic uncertain, and state. Ordering of variable types below this granularity (e.g., from normal to histogram bin within continuous aleatory uncertain) is defined somewhat arbitrarily, but is enforced consistently throughout the code.

Several examples follow. In the first example, two continuous design variables are specified:

```
variables,
continuous_design = 2
  initial_point 0.9  1.1
  upper_bounds  5.8  2.9
  lower_bounds  0.5 -2.9
  descriptors 'radius' 'location'
```

In the next example, defaults are employed. In this case, `initial_point` will default to a vector of 0. values, `upper_bounds` will default to vector values of `DBL_MAX` (the maximum number representable in double precision for a particular platform, as defined in the platform's `float.h` C header file), `lower_bounds` will default to a vector of `-DBL_MAX` values, and `descriptors` will default to a vector of `'cdv_i'` strings, where `i` ranges from one to two:

```
variables,
continuous_design = 2
```

In the following example, the syntax for a normal-lognormal distribution is shown. One normal and one lognormal uncertain variable are completely specified by their means and standard deviations. In addition, the dependence structure between the two variables is specified using the `uncertain_correlation_matrix`.

```
variables,
  normal_uncertain = 1
  means = 1.0
  std_deviations = 1.0
  descriptors = 'TF1n'
  lognormal_uncertain = 1
  means = 2.0
```

An example of the syntax for a state variables specification follows:

variables,
  continuous_state = 1
  initial_state 4.0
  lower_bounds 0.0
  upper_bounds 8.0
  descriptors 'CS1'
  discrete_state_range = 1
  initial_state 104
  lower_bounds 100
  upper_bounds 110
  descriptors 'DS1'

And in a more advanced example, a variables specification containing a set identifier, continuous and discrete design variables, normal and uniform uncertain variables, and continuous and discrete state variables is shown:

variables,
  id_variables = 'V1'
  continuous_design = 2
  initial_point 0.9 1.1
  upper_bounds 5.8 2.9
  lower_bounds 0.5 -2.9
  descriptors 'radius' 'location'
  discrete_design_range = 1
  initial_point 2
  upper_bounds 1
  lower_bounds 3
  descriptors 'material'
  normal_uncertain = 2
  means = 248.89, 593.33
  std_deviations = 12.4, 29.7
  descriptors = 'TF1n' 'TF2n'
  uniform_uncertain = 2
  lower_bounds = 199.3, 474.63
  upper_bounds = 298.5, 712.
  descriptors = 'TF1u' 'TF2u'
  continuous_state = 2
  initial_state = 1.e-4 1.e-6
  descriptors = 'EPSIT1' 'EPSIT2'
  discrete_state_set_int = 1
  initial_state = 100
  set_values = 100 212 375
  descriptors = 'load_case'

Refer to the DAKOTA Users Manual [Adams et al., 2010] for discussion on how different iterators view these mixed variable sets.
variables,
<set identifier>
<continuous design variables specification>
<discrete design range variables specification>
<discrete design set integer variables specification>
<discrete design set real variables specification>
<normal uncertain variables specification>
<lognormal uncertain variables specification>
<uniform uncertain variables specification>
<loguniform uncertain variables specification>
<triangular uncertain variables specification>
<exponential uncertain variables specification>
<beta uncertain variables specification>
<gamma uncertain variables specification>
<gumbel uncertain variables specification>
<frechet uncertain variables specification>
<weibull uncertain variables specification>
<histogram bin uncertain variables specification>
<poisson uncertain variables specification>
<binomial uncertain variables specification>
<negative binomial uncertain variables specification>
<geometric uncertain variables specification>
<hypergeometric uncertain variables specification>
<histogram point uncertain variables specification>
<uncertain correlation specification>
<interval uncertain variables specification>
<continuous state variables specification>
<discrete state range variables specification>
<discrete state set integer variables specification>
<discrete state set real variables specification>

Referring to dakota.input.summary, it is evident from the enclosing brackets that the set identifier specification, the uncertain correlation specification, and each of the variables specifications are all optional. The set identifier and uncertain correlation are stand-alone optional specifications, whereas the variables specifications are optional group specifications, meaning that the group can either appear or not as a unit. If any part of an optional group is specified, then all required parts of the group must appear.

The optional status of the different variable type specifications allows the user to specify only those variables which are present (rather than explicitly specifying that the number of a particular type of variables is zero). However, at least one type of variables that are active for the iterator in use must have nonzero size or an input error message will result. The following sections describe each of these specification components in additional detail.

7.3 Variables Set Identifier

The optional set identifier specification uses the keyword id_variables to input a unique string for use in identifying a particular variables set. A model can then identify the use of this variables set by specifying the same string in its variables_pointer specification (see Model Independent Controls). For example, a model whose specification contains variables_pointer = ‘V1’ will use a variables specification containing the set identifier id_variables = ‘V1’.

If the id_variables specification is omitted, a particular variables set will be used by a model only if that model omits specifying a variables_pointer and if the variables set was the last set parsed (or is the only set parsed). In common practice, if only one variables set exists, then id_variables can be safely omitted from the variables specification and variables_pointer can be omitted from the model specification(s).
since there is no potential for ambiguity in this case. Table 7.1 summarizes the set identifier inputs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables set identifier</td>
<td>id_variables</td>
<td>string</td>
<td>Optional</td>
<td>use of last variables parsed</td>
</tr>
</tbody>
</table>

Table 7.1: Specification detail for set identifier

7.4 Design Variables

Design variable types include continuous real, discrete range of integer values (contiguous integers), discrete set of integer values, and discrete set of real values. Within each optional design variables specification group, the number of variables is always required. The following Tables 7.2 through 7.5 summarize the required and optional specifications for each design variable subtype. The initial_point specifications provide the point in design space from which an iterator is started and default to either zeros (continuous and discrete range) or middle values (discrete sets). The descriptors specifications supply strings which will be replicated through the DAKOTA output to identify the numerical values for these parameters; these default to numbered strings.

For continuous and discrete range variables, the lower_bounds and upper_bounds restrict the size of the feasible design space and are frequently used to prevent nonphysical designs. Default values are positive and negative machine limits for upper and lower bounds (+/- DBL_MAX, INT_MAX, INT_MIN from the float.h and limits.h system header files). As for linear and nonlinear inequality constraint bounds (see Method Independent Controls and Objective and constraint functions (optimization data set)), a nonexistent upper bound can be specified by using a value greater than the “big bound size” constant (1.e+30 for continuous variables, 1.e+9 for discrete integer variables) and a nonexistent lower bound can be specified by using a value less than the negation of these constants (-1.e+30 for continuous, -1.e+9 for discrete integer), although not all optimizers currently support this feature (e.g., DOT and CONMIN will treat these large bound values as actual variable bounds, but this should not be problematic in practice).

7.4.1 Continuous Design Variables

For continuous variables, the scale_types specification includes strings specifying the scaling type for each component of the continuous design variables vector in methods that support scaling, when scaling is enabled (see Method Independent Controls for details). Each entry in scale_types may be selected from ‘none’, ‘value’, ‘auto’, or ‘log’, to select no, characteristic value, automatic, or logarithmic scaling, respectively. If a single string is specified it will apply to all components of the continuous design variables vector. Each entry in scales may be a user-specified nonzero real characteristic value to be used in scaling each variable component. These values are ignored for scaling type ‘none’, required for ‘value’, and optional for ‘auto’ and ‘log’. If a single real value is specified it will apply to all components of the continuous design variables vector.

7.4.2 Discrete Design Range Variables

7.4.3 Discrete Design Integer Set Variables

Discrete set variables are specified with an integer list specifying how many set members there are for each variable and a list of integer or real set values for discrete_design_set_integer (Table 7.4) and discrete_design_set_real (Table 7.5), respectively.
### Table 7.2: Specification detail for continuous design variables

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous design variables</td>
<td>continuous_design</td>
<td>integer</td>
<td>Optional group</td>
<td>no continuous design variables</td>
</tr>
<tr>
<td>Initial point</td>
<td>initial_point</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 0. (repaired to bounds, if required)</td>
</tr>
<tr>
<td>Lower bounds</td>
<td>lower_bounds</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = -DBL_MAX</td>
</tr>
<tr>
<td>Upper bounds</td>
<td>upper_bounds</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = +DBL_MAX</td>
</tr>
<tr>
<td>Scaling types</td>
<td>scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
</tr>
<tr>
<td>Scales</td>
<td>scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 1. (no scaling)</td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of 'cdv_i' where i = 1,2,3...</td>
</tr>
</tbody>
</table>

### Table 7.3: Specification detail for discrete design range variables

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete design range variables</td>
<td>discrete_design_range</td>
<td>integer</td>
<td>Optional group</td>
<td>no discrete design variables</td>
</tr>
<tr>
<td>Initial point</td>
<td>initial_point</td>
<td>list of integers</td>
<td>Optional</td>
<td>vector values = 0. (repaired to bounds, if required)</td>
</tr>
<tr>
<td>Lower bounds</td>
<td>lower_bounds</td>
<td>list of integers</td>
<td>Optional</td>
<td>vector values = INT_MIN</td>
</tr>
<tr>
<td>Upper bounds</td>
<td>upper_bounds</td>
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</table>
7.5 Aleatory Uncertain Variables

Aleatory uncertain variables involve continuous or discrete probability distribution specifications. Continuous probability distributions including normal, lognormal, uniform, loguniform, triangular, exponential, beta, gamma, gumbel, frechet, weibull, and histogram bin distributions. Discrete probability distributions include poisson, binomial, negative binomial, geometric, hypergeometric, and histogram point distributions. Each of these specifications is an optional group specification.

These specifications of probability distributions directly support the use of probabilistic uncertainty quantification methods such as sampling, reliability, and stochastic expansion methods. However, the inclusion of lower and upper distribution bounds for all uncertain variable types (either explicitly defined, implicitly defined, or inferred;
see Variables Description) also allows the use of these variables within methods that rely on a bounded region to define a set of function evaluations (i.e., design of experiments and some parameter study methods). Each distribution also provides optional uncertain variable descriptors (default values are numbered strings) that supply identifiers that help associate the numerical values with the uncertain parameters as they appear within the DAKOTA output. Tables 7.6 through 7.23 summarize the details of the aleatory uncertain variable specifications.

7.5.1 Normal Distribution

Within the normal uncertain optional group specification, the number of normal uncertain variables, the means, and standard deviations are required specifications, and the distribution lower and upper bounds and variable descriptors are optional specifications. The normal distribution is widely used to model uncertain variables such as population characteristics. It is also used to model the mean of a sample: as the sample size becomes very large, the Central Limit Theorem states that the mean becomes approximately normal, regardless of the distribution of the original variables.

The density function for the normal distribution is:

\[
f(x) = \frac{1}{\sqrt{2\pi \sigma_N}} e^{-\frac{1}{2} \left( \frac{x-\mu_N}{\sigma_N} \right)^2}
\]

where \( \mu_N \) and \( \sigma_N \) are the mean and standard deviation of the normal distribution, respectively.

Note that if you specify bounds for a normal distribution, the sampling occurs from the underlying distribution with the given mean and standard deviation, but samples are not taken outside the bounds (see "bounded normal" distribution type in [Wyss and Jorgensen, 1998]). This can result in the mean and the standard deviation of the sample data being different from the mean and standard deviation of the underlying distribution. For example, if you are sampling from a normal distribution with a mean of 5 and a standard deviation of 3, but you specify bounds of 1 and 7, the resulting mean of the samples will be around 4.3 and the resulting standard deviation will be around 1.6. This is because you have bounded the original distribution significantly, and asymmetrically, since 7 is closer to the original mean than 1.

<table>
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Table 7.6: Specification detail for normal uncertain variables
7.5.2 Lognormal Distribution

If the logarithm of an uncertain variable $X$ has a normal distribution, that is $\log X \sim N(\mu, \sigma)$, then $X$ is distributed with a lognormal distribution. The lognormal is often used to model time to perform some task. It can also be used to model variables which are the product of a large number of other quantities, by the Central Limit Theorem. Finally, the lognormal is used to model quantities which cannot have negative values. Within the lognormal uncertain optional group specification, the number of lognormal uncertain variables, the means, and either standard deviations or error factors must be specified, and the distribution lower and upper bounds and variable descriptors are optional specifications. These distribution bounds can be used to truncate the tails of lognormal distributions, which as for bounded normal, can result in the mean and the standard deviation of the sample data being different from the mean and standard deviation of the underlying distribution (see "bounded lognormal" and "bounded lognormal-n" distribution types in [Wyss and Jorgensen, 1998]).

For the lognormal variables, one may specify either the mean $\mu$ and standard deviation $\sigma$ of the actual lognormal distribution, the mean $\mu$ and error factor $\epsilon$ of the actual lognormal distribution, or the mean $\lambda$ (“lambda”) and standard deviation $\zeta$ (“zeta”) of the underlying normal distribution. The conversion equations from lognormal mean $\mu$ and either lognormal error factor $\epsilon$ or lognormal standard deviation $\sigma$ to the mean $\lambda$ and standard deviation $\zeta$ of the underlying normal distribution are as follows:

\[
\zeta = \frac{\ln(\epsilon)}{1.645}
\]
\[
\zeta^2 = \ln\left(\frac{\sigma^2}{\mu^2} + 1\right)
\]
\[
\lambda = \ln(\mu) - \frac{\zeta^2}{2}
\]

Conversions from $\lambda$ and $\zeta$ back to $\mu$ and $\epsilon$ or $\sigma$ are as follows:

\[
\mu = e^{\lambda + \zeta^2}
\]
\[
\sigma^2 = e^{2\lambda + \zeta^2} (e^{\zeta^2} - 1)
\]
\[
\epsilon = e^{1.645\zeta}
\]

The density function for the lognormal distribution is:

\[
f(x) = \frac{1}{\sqrt{2\pi}\zeta x} e^{-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2}
\]

7.5.3 Uniform Distribution

Within the uniform uncertain optional group specification, the number of uniform uncertain variables and the distribution lower and upper bounds are required specifications, and variable descriptors is an optional specification. The uniform distribution has the density function:

\[
f(x) = \frac{1}{U_U - L_U}
\]

where $U_U$ and $L_U$ are the upper and lower bounds of the uniform distribution, respectively. The mean of the uniform distribution is $\frac{U_U + L_U}{2}$ and the variance is $\frac{(U_U - L_U)^2}{12}$. Note that this distribution is a special case of the more general beta distribution.
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Table 7.7: Specification detail for lognormal uncertain variables

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<td>vector of ‘uuv_i’ where i = 1,2,3,...</td>
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</table>

Table 7.8: Specification detail for uniform uncertain variables
7.5.4 Loguniform Distribution

If the logarithm of an uncertain variable $X$ has a uniform distribution, that is $\log X \sim U(L_{LU}, U_{LU})$, then $X$ is distributed with a loguniform distribution. Within the loguniform uncertain optional group specification, the number of loguniform uncertain variables and the distribution lower and upper bounds are required specifications, and variable descriptors is an optional specification. The loguniform distribution has the density function:

$$f(x) = \frac{1}{x(lnU_{LU} - lnL_{LU})}$$

<table>
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<th>Associated Data</th>
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Table 7.9: Specification detail for loguniform uncertain variables

7.5.5 Triangular Distribution

The triangular distribution is often used when one does not have much data or information, but does have an estimate of the most likely value and the lower and upper bounds. Within the triangular uncertain optional group specification, the number of triangular uncertain variables, the modes, and the distribution lower and upper bounds are required specifications, and variable descriptors is an optional specification.

The density function for the triangular distribution is:

$$f(x) = \frac{2(x - L_T)}{(U_T - L_T)(M_T - L_T)}$$

if $L_T \leq x \leq M_T$, and

$$f(x) = \frac{2(U_T - x)}{(U_T - L_T)(U_T - M_T)}$$

if $M_T \leq x \leq U_T$, and 0 elsewhere. In these equations, $L_T$ is the lower bound, $U_T$ is the upper bound, and $M_T$ is the mode of the triangular distribution.

7.5.6 Exponential Distribution

The exponential distribution is often used for modeling failure rates. Within the exponential uncertain optional group specification, the number of exponential uncertain variables and the beta parameters are required specifications, and variable descriptors is an optional specification.
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</table>

Table 7.10: Specification detail for triangular uncertain variables

The density function for the exponential distribution is given by:

\[
f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}
\]

where \(\mu_E = \beta\) and \(\sigma_E^2 = \beta^2\). Note that this distribution is a special case of the more general gamma distribution.

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Table 7.11: Specification detail for exponential uncertain variables

### 7.5.7 Beta Distribution

Within the beta uncertain optional group specification, the number of beta uncertain variables, the alpha and beta parameters, and the distribution upper and lower bounds are required specifications, and the variable descriptors is an optional specification. The beta distribution can be helpful when the actual distribution of an uncertain variable is unknown, but the user has a good idea of the bounds, the mean, and the standard deviation of the uncertain variable. The density function for the beta distribution is

\[
f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(x - L_B)^{\alpha-1}(U_B - x)^{\beta-1}}{(U_B - L_B)^{\alpha+\beta-1}}
\]

where \(\Gamma(\alpha)\) is the gamma function and \(B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\) is the beta function. To calculate mean and standard deviation from the alpha, beta, upper bound, and lower bound parameters of the beta distribution, the following expressions may be used.

\[
\mu_B = L_B + \frac{\alpha}{\alpha + \beta}(U_B - L_B)
\]
\[ \sigma_B^2 = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)} (U_B - L_B)^2 \]

Solving these for \( \alpha \) and \( \beta \) gives:

\[ \alpha = (\mu_B - L_B) \frac{(\mu_B - L_B)(U_B - \mu_B) - \sigma_B^2}{\sigma_B^2 (U_B - L_B)} \]

\[ \beta = (U_B - \mu_B) \frac{(\mu_B - L_B)(U_B - \mu_B) - \sigma_B^2}{\sigma_B^2 (U_B - L_B)} \]

Note that the uniform distribution is a special case of this distribution for parameters \( \alpha = \beta = 1 \).

<table>
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Table 7.12: Specification detail for beta uncertain variables

7.5.8 Gamma Distribution

The gamma distribution is sometimes used to model time to complete a task, such as a repair or service task. It is a very flexible distribution. Within the gamma uncertain optional group specification, the number of gamma uncertain variables and the alpha and beta parameters are required specifications, and variable descriptors is an optional specification.

The density function for the gamma distribution is given by:

\[ f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} \]

where \( \mu_{GA} = \alpha \beta \) and \( \sigma_{GA}^2 = \alpha \beta^2 \). Note that the exponential distribution is a special case of this distribution for parameter \( \alpha = 1 \).

7.5.9 Gumbel Distribution

Within the gumbel optional uncertain group specification, the number of gumbel uncertain variables, and the alpha and beta parameters are required specifications. The Gumbel distribution is also referred to as the Type I
Largest Extreme Value distribution. The distribution of maxima in sample sets from a population with a normal distribution will asymptotically converge to this distribution. It is commonly used to model demand variables such as wind loads and flood levels.

The density function for the Gumbel distribution is given by:

\[ f(x) = \alpha e^{-\alpha(x-\beta)} \exp(-e^{-\alpha(x-\beta)}) \]

where \( \mu_{GU} = \beta + \frac{0.5772}{\alpha} \) and \( \sigma_{GU} = \frac{\pi}{\sqrt{6\alpha}} \).

---

### Frechet Distribution

With the frechet uncertain optional group specification, the number of frechet uncertain variables and the alpha and beta parameters are required specifications. The Frechet distribution is also referred to as the Type II Largest Extreme Value distribution. The distribution of maxima in sample sets from a population with a lognormal distribution will asymptotically converge to this distribution. It is commonly used to model non-negative demand variables.

The density function for the frechet distribution is:

\[ f(x) = \frac{\alpha}{\beta} \left( \frac{\beta}{x} \right)^\alpha e^{-\left( \frac{\beta}{x} \right)^\alpha} \]
where \( \mu_F = \beta \Gamma(1 - \frac{1}{\alpha}) \) and \( \sigma^2_F = \beta^2 [\Gamma(1 - \frac{2}{\alpha}) - \Gamma^2(1 - \frac{1}{\alpha})] \)

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Table 7.15: Specification detail for frechet uncertain variables

### 7.5.11 Weibull Distribution

The Weibull distribution is commonly used in reliability studies to predict the lifetime of a device. Within the weibull uncertain optional group specification, the number of weibull uncertain variables and the alpha and beta parameters are required specifications. The Weibull distribution is also referred to as the Type III Smallest Extreme Value distribution. It is also used to model capacity variables such as material strength.

The density function for the weibull distribution is given by:

\[
f(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)\alpha}
\]

where \( \mu_W = \beta \Gamma(1 + \frac{1}{\alpha}) \) and \( \sigma_W = \sqrt{\frac{\Gamma(1+\frac{2}{\alpha})}{\Gamma^2(1+\frac{1}{\alpha})} - 1} \mu_W \)

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Table 7.16: Specification detail for weibull uncertain variables

### 7.5.12 Histogram Bin Distribution

Histogram uncertain variables are typically used to model a set of empirical data. A bin histogram is a continuous aleatory distribution that allows the user to specify bins of non-zero width (where the uncertain variable may lie) along with the relative frequencies that are associated with each bin.
Within the histogram bin uncertain optional group specification, the number of histogram bin uncertain variables is a required specification, the number of pairs is an optional key for apportionment of abscissas/ordinates/counts, specification of abscissas and either ordinates or counts is required, and the variable descriptors is an optional specification. When using a histogram bin variable, one must define at least one bin (with two bounding value pairs).

The **abscissas** specification define abscissa values ("x" coordinates) for the PDF of each histogram variable. When paired with **counts**, the specifications provide sets of (x,c) pairs for each histogram variable where c defines a count (i.e., a frequency or relative probability) associated with a bin. If using bins of unequal width and specification of probability densities is more natural, then the **counts** specification can be replaced with a **ordinates** specification ("y" coordinates) in order to support interpretation of the input as (x,y) pairs defining the profile of a "skyline" PDF. Conversion between the two specifications is straightforward: a count/frequency is a cumulative probability quantity defined from the product of the ordinate density value and the x bin width. Thus, in the cases of bins of equal width, ordinate and count specifications are equivalent. In addition, ordinates and counts may be relative values; it is not necessary to scale them as all user inputs will be normalized.

To fully specify a bin-based histogram with n bins (potentially of unequal width), n+1 (x,c) or (x,y) pairs must be specified with the following features:

- x is the parameter value for the left boundary of a histogram bin and c is the corresponding count for that bin. Alternatively, y defines the ordinate density value for this bin within a skyline PDF. The right boundary of the bin is defined by the left boundary of the next pair.
- the final pair specifies the right end of the last bin and must have a c or y value of zero.
- the x values must be strictly increasing.
- all c or y values must be positive, except for the last which must be zero.
- a minimum of two pairs must be specified for each bin-based histogram.

The number of pairs specifications provide for the proper association of multiple sets of (x,c) or (x,y) pairs with individual histogram variables. For example, in the following specification

```
histogram_bin_uncertain = 2
  num_pairs = 3 4
  abscissas = 5 8 10 .1 .2 .3 .4
  counts = 17 21 0 12 24 12 0
```

**num_pairs** associates the first 3 (x,c) pairs from **abscissas** and **counts** ((5,17),(8,21),(10,0)) with one bin-based histogram variable, where one bin is defined between 5 and 8 with a count of 17 and another bin is defined between 8 and 10 with a count of 21. The following set of 4 (x,c) pairs ((.1,12),(.2,24),(.3,12),(.4,0)) defines a second bin-based histogram variable containing three equal-width bins with counts 12, 24, and 12 (middle bin is twice as probable as the other two).

### 7.5.13 Poisson Distribution

The Poisson distribution is used to predict the number of discrete events that happen in a given time interval. The expected number of occurrences in the time interval is \( \lambda \), which must be a positive real number. For example, if events occur on average 4 times per year and we are interested in the distribution of events over six months, \( \lambda \)
### Table 7.17: Specification detail for histogram bin uncertain variables

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
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<tbody>
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<td>histogram_bin_</td>
<td>integer</td>
<td>Optional group</td>
<td>no histogram bin uncertain variables</td>
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<tr>
<td></td>
<td>uncertain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>key to apportionment among bin-based histogram</td>
<td>num_pairs</td>
<td>list of integers</td>
<td>Optional</td>
<td>equal distribution</td>
</tr>
<tr>
<td>variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sets of abscissas for bin-based histogram</td>
<td>abscissas</td>
<td>list of reals</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sets of ordinates for bin-based histogram</td>
<td>ordinates</td>
<td>list of reals</td>
<td>Required (1 of 2</td>
<td>N/A</td>
</tr>
<tr>
<td>variables</td>
<td></td>
<td></td>
<td>selections)</td>
<td></td>
</tr>
<tr>
<td>sets of counts for bin-based histogram variables</td>
<td>counts</td>
<td>list of reals</td>
<td>Required (1 of 2</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>selections)</td>
<td></td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ('hubv_i' where i = 1,2,3,...)</td>
</tr>
</tbody>
</table>

Table 7.17: Specification detail for histogram bin uncertain variables
would be 2 in this case. However, if we were interested in the distribution of events occurring over 5 years, \( \lambda \) would be 20.

The density function for the poisson distribution is given by:

\[
f(x) = \frac{\lambda e^{-\lambda}}{x!}
\]

where \( \lambda \) is the frequency of events happening, and \( x \) is the number of events that occur. The poisson distribution returns samples representing number of occurrences in the time period of interest.

<table>
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<tr>
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<td>uncertain</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>lambdas</td>
<td>list of reals</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>lambdas</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ('p_{uv_i}') where ( i = 1,2,3,... )</td>
</tr>
</tbody>
</table>

Table 7.18: Specification detail for poisson uncertain variables

7.5.14 Binomial Distribution

The binomial distribution is typically used to predict the number of failures (or defective items or some type of event) in a total of \( n \) independent tests or trials, where each trial has the probability \( p \) of failing or being defective. Each particular test can be considered as a Bernoulli trial.

The density function for the binomial distribution is given by:

\[
f(x) = \binom{n}{x} p^x (1-p)^{n-x}
\]

where \( p \) is the probability of failure per trial and \( n \) is the number of trials.

<table>
<thead>
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<th>Associated Data</th>
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<tr>
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<td></td>
<td></td>
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<td>prob_per_</td>
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<td>N/A</td>
</tr>
<tr>
<td>prob_per_trial</td>
<td>trial</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>list of integers</td>
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<tr>
<td>num_trials</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ('biuv_i') where ( i = 1,2,3,... )</td>
</tr>
</tbody>
</table>

Table 7.19: Specification detail for binomial uncertain variables
7.5.15 Negative Binomial Distribution

The negative binomial distribution is typically used to predict the number of times to perform a test to have a total of \( n \) successes, where each test has a probability \( p \) of success.

The density function for the negative binomial distribution is given by:

\[
f(x) = \binom{n + x - 1}{x} p^n (1 - p)^x
\]

where \( p \) is the probability of success per trial and \( n \) is the number of successful trials.

<table>
<thead>
<tr>
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<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
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<td>prob_per_trial</td>
<td>list of reals</td>
<td>Required</td>
<td>N/A</td>
</tr>
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<td>num_trials</td>
<td>list of integers</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of 'nbuv_i' where ( i = 1, 2, 3, \ldots )</td>
</tr>
</tbody>
</table>
### Variables Commands

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<td>N/A</td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ‘geuv_i’ where i = 1,2,3,...</td>
</tr>
</tbody>
</table>

Table 7.21: Specification detail for geometric uncertain variables

The density function for the hypergeometric distribution is given by:

\[
f(x) = \binom{m}{x} \binom{N-m}{n-x} / \binom{N}{n}
\]

where N is the total population, m is the number of items in the selected population (e.g. the number of white balls in the full urn of N items), and n is the number of balls drawn.

<table>
<thead>
<tr>
<th>Description</th>
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<th>Status</th>
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</tr>
</thead>
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<td>Optional group</td>
<td>no hypergeometric uncertain variables</td>
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<td>N/A</td>
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<tr>
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<td>list of integers</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ‘hguv_i’ where i = 1,2,3,...</td>
</tr>
</tbody>
</table>

Table 7.22: Specification detail for hypergeometric uncertain variables

**7.5.18 Histogram Point Distribution**

As mentioned above, histogram uncertain variables are typically used to model a set of empirical data. A point histogram is a discrete aleatory distribution that allows the user to specify a set of real-valued points and associated frequency values.
Point histograms are similar to Discrete Design Real Set Variables and Discrete State Real Set Variables, but differ in the inclusion of information on the relative probabilities of observing the different values within the set.

Within the histogram point uncertain optional group specification, the number of histogram point uncertain variables is a required specification, the number of pairs is an optional key for apportionment of abscissas and counts, the sets of abscissas and counts are required, and the variable descriptors are optional. When using a histogram point variable, one must define at least one set of abscissa/count pairs. As for Histogram Bin Distribution, the abscissas specifications define abscissa values ("x" coordinates) for the PDF of each histogram variable. When paired with counts, the specifications provide sets of (x,c) pairs for each histogram variable where c defines a count (i.e., a frequency or relative probability) associated with a point.

To fully specify a point-based histogram with n points, n (x,c) pairs (note that (x,c) and (x,y) are equivalent in this case) must be specified with the following features:

- x is the point value and c is the corresponding count for that value.
- the x values must be strictly increasing.
- all c values must be positive.
- a minimum of one pair must be specified for each point-based histogram.

The num_pairs specification provides for the proper association of multiple sets of (x,c) or (x,y) pairs with individual histogram variables. For example, in the following specification,

```
histogram_point_uncertain = 2
num_pairs = 2 3
abscissas = 3 4 100 200 300
counts = 1 1 1 2 1
```

num_pairs associates the (x,c) pairs ((3,1),(4,1)) with one point-based histogram variable (where the values 3 and 4 are equally probable) and associates the (x,c) pairs ((100,1),(200,2),(300,1)) with a second point-based histogram variable (where the value 200 is twice as probable as either 100 or 300).

7.5.19 Correlations

Aleatory uncertain variables may have correlations specified through use of an uncertain_correlation_matrix specification. This specification is generalized in the sense that its specific meaning depends on the non-deterministic method in use. When the method is a nondeterministic sampling method (i.e., sampling), then the correlation matrix specifies rank correlations [Iman and Conover, 1982]. When the method is instead a reliability (i.e., local_reliability or global_reliability) or stochastic expansion (i.e., polynomial_chaos or stoch_collocation) method, then the correlation matrix specifies correlation coefficients (normalized covariance) [Haldar and Mahadevan, 2000]. In either of these cases, specifying the identity matrix results in uncorrelated uncertain variables (the default). The matrix input should be symmetric and have all n² entries where n is the total number of aleatory uncertain variables (all normal, lognormal, uniform, loguniform, triangular, exponential, beta, gamma, gumbel, frechet, weibull, histogram bin, poisson, binomial, negative binomial, geometric, hypergeometric, and histogram point specifications, in that order). Table 7.24 summarizes the specification details:
<table>
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<tr>
<th>Description</th>
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<th>Associated Data</th>
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</tr>
</thead>
<tbody>
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<td>histogram_point_uncertain</td>
<td>integer</td>
<td>Optional group</td>
<td>no histogram point uncertain variables</td>
</tr>
<tr>
<td>key to apportionment among point-based histogram variables</td>
<td>num_pairs</td>
<td>list of integers</td>
<td>Optional</td>
<td>equal distribution</td>
</tr>
<tr>
<td>sets of abscissas for point-based histogram variables</td>
<td>abscissas</td>
<td>list of reals</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>sets of counts for point-based histogram variables</td>
<td>counts</td>
<td>list of reals</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ‘hupv_i’ where i = 1,2,3,...</td>
</tr>
</tbody>
</table>

Table 7.23: Specification detail for histogram point uncertain variables

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
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<td>uncertain_correlation_matrix</td>
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<td>Optional</td>
<td>identity matrix (uncorrelated)</td>
</tr>
</tbody>
</table>

Table 7.24: Specification detail for aleatory uncertain correlations
7.6 Epistemic Uncertain Variables

In addition to continuous and discrete aleatory probability distributions, DAKOTA provides support for epistemic uncertainties through its interval variable specification. This is not a probability distribution; rather, it specifies a set of belief structures based on intervals that may be contiguous, overlapping, or disjoint. It is used in specifying the inputs necessary for an epistemic uncertainty analysis using Dempster-Shafer theory of evidence.

7.6.1 Interval Uncertain Variable

The interval uncertain variable is NOT a probability distribution. Although it may seem similar to a histogram, the interpretation of this uncertain variable is different. It is used in epistemic uncertainty analysis, where one is trying to model uncertainty due to lack of knowledge. In DAKOTA, epistemic uncertainty analysis is performed using Dempster-Shafer theory of evidence. In this approach, one does not assign a probability distribution to each uncertain input variable. Rather, one divides each uncertain input variable into one or more intervals. The input parameters are only known to occur within intervals: nothing more is assumed. Each interval is defined by its upper and lower bounds, and a Basic Probability Assignment (BPA) associated with that interval. The BPA represents a probability of that uncertain variable being located within that interval. The intervals and BPAs are used to construct uncertainty measures on the outputs called "belief" and "plausibility." Belief represents the smallest possible probability that is consistent with the evidence, while plausibility represents the largest possible probability that is consistent with the evidence. For more information about the Dempster-Shafer approach, see the nondeterministic evidence method, evidence, in the Methods section of this Reference manual. As an example, in the following specification:

```plaintext
interval_uncertain = 2
num_intervals = 3 2
interval_probs = 0.2 0.5 0.3 0.4 0.6
interval_bounds = 2.0 2.5 4.0 5.0 4.5 6.0 1.0 5.0 3.0 5.0
```

there are 2 interval uncertain variables. The first one is defined by three intervals, and the second by two intervals. The three intervals for the first variable have basic probability assignments of 0.2, 0.5, and 0.3, respectively, while the basic probability assignments for the two intervals for the second variable are 0.4 and 0.6. The basic probability assignments for each interval variable must sum to one. The interval bounds for the first variable are [2, 2.5], [4, 5], and [4.5, 6], and the interval bounds for the second variable are [1, 5] and [3, 5].

Note that the intervals can be overlapping or disjoint. Table 7.25 summarizes the specification details for the interval_uncertain variable.

7.7 State Variables

State variables provide a convenient mechanism for managing additional model parameterizations such as mesh density, simulation convergence tolerances, and time step controls. Types include continuous real, discrete range of integer values (contiguous integers), discrete set of integer values, and discrete set of real values. Within each optional state variables specification group, the number of variables is always required. The following Tables 7.26 through 7.29 summarize the required and optional specifications for each state variable subtype. The initial_state specifications provide the initial values for the state variables which will be passed through to the simulator (e.g., in order to define parameterized modeling controls). The remaining specifications are analogous to those for Design Variables.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
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<td>no interval uncertain variables</td>
</tr>
<tr>
<td>number of intervals defined for each interval variable</td>
<td><code>num_ - intervals</code></td>
<td>list of integers</td>
<td>Required group</td>
<td>None</td>
</tr>
<tr>
<td>basic probability assignments per interval</td>
<td><code>interval_ - probs</code></td>
<td>list of reals</td>
<td>Required group. Note that the probabilities per variable must sum to one.</td>
<td>None</td>
</tr>
<tr>
<td>bounds per interval</td>
<td><code>interval_ - bounds</code></td>
<td>list of reals</td>
<td>Required group. Specify bounds as (lower, upper) per interval, per variable</td>
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</tr>
<tr>
<td>Descriptors</td>
<td><code>descriptors</code></td>
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<td>Optional</td>
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</table>

Table 7.25: Specification detail for interval uncertain variables

### 7.7.1 Continuous State Variables

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<td>Initial states</td>
<td><code>initial_ - state</code></td>
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</tr>
<tr>
<td>Lower bounds</td>
<td><code>lower_bounds</code></td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = -DBL_MAX</td>
</tr>
<tr>
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<td>list of reals</td>
<td>Optional</td>
<td>vector values = +DBL_MAX</td>
</tr>
<tr>
<td>Descriptors</td>
<td><code>descriptors</code></td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ‘csv_i’ where i = 1,2,3,...</td>
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</table>

Table 7.26: Specification detail for continuous state variables

### 7.7.2 Discrete State Range Variables

### 7.7.3 Discrete State Integer Set Variables

### 7.7.4 Discrete State Real Set Variables
### Table 7.27: Specification detail for discrete state range variables

<table>
<thead>
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<th>Description</th>
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<td>initial_state</td>
<td>list of integers</td>
<td>Optional</td>
<td>vector values = 0 (repaired to bounds, if required)</td>
</tr>
<tr>
<td>Lower bounds</td>
<td>lower_bounds</td>
<td>list of integers</td>
<td>Optional</td>
<td>vector values = INT_MIN</td>
</tr>
<tr>
<td>Upper bounds</td>
<td>upper_bounds</td>
<td>list of integers</td>
<td>Optional</td>
<td>vector values = INT_MAX</td>
</tr>
<tr>
<td>Descriptors</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ‘dsriv_i’ where i = 1, 2, 3,...</td>
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</table>

### Table 7.28: Specification detail for discrete state set of integer variables

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<td>middle set values (mean indices, rounded down)</td>
</tr>
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<td>Number of values for each variable</td>
<td>num_set_values</td>
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<td>Optional</td>
<td>equal distribution</td>
</tr>
<tr>
<td>Set values</td>
<td>set_values</td>
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<td>Required</td>
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</tr>
<tr>
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<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector of ‘dssiv_i’ where i = 1, 2, 3,...</td>
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<td>list of integers</td>
<td>Optional</td>
<td>equal distribution</td>
</tr>
<tr>
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<td><code>set_values</code></td>
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<td>list of strings</td>
<td>Optional</td>
<td>vector of ‘dssrv_i’ where i = 1,2,3,...</td>
</tr>
</tbody>
</table>

Table 7.29: Specification detail for discrete state set of real variables
Chapter 8

Interface Commands

8.1 Interface Description

The interface section in a DAKOTA input file specifies how function evaluations will be performed in order to map a set of parameters into a set of responses. Function evaluations are performed using either algebraic mappings, interfaces to simulation codes, or a combination of the two.

When employing algebraic mappings, the AMPL solver library [Gay, 1997] is used to evaluate a directed acyclic graph (DAG) specification from a separate stub.nl file. Separate stub.col and stub.row files are also required to declare the string identifiers of the subset of inputs and outputs, respectively, that will be used in the algebraic mappings.

When employing mappings with simulation codes, the simulations may be available internally or externally to DAKOTA. The interface invokes the simulation using either system calls, forks, direct function invocations, or computational grid invocations. In the system call and fork cases, the simulation is external to DAKOTA and communication between DAKOTA and the simulation occurs through parameter and response files. In the direct function case, the simulation is internal to DAKOTA and communication occurs through the function parameter list. The direct case can involve linked simulation codes or test functions which are compiled into the DAKOTA executable. The test functions allow for rapid testing of algorithms without process creation overhead or engineering simulation expense. The grid case is experimental and under development, but is intended to support simulations which are external to DAKOTA and geographically distributed.

Several examples follow. The first example shows a fork interface specification which specifies the names of the analysis executable and the parameters and results files, and that parameters and responses files will be tagged and saved. Refer to Fork interface for more information on the use of these options.

```
interface,  
fork  
  analysis_drivers = 'rosenbrock'  
  parameters_file = 'params.in'  
  results_file = 'results.out'  
  file_tag  
  file_save
```

The next example shows a similar specification, except that an external rosenbrock executable has been replaced by use of the internal rosenbrock test function from the DirectApplicInterface class. Refer to Direct function interface for more information on this specification.
The final example demonstrates an interface employing both algebraic and simulation-based mappings. The results from the individual mappings are overlaid based on the variable and response descriptors used by the individual mappings.

interface,
direct
   analysis_drivers = 'rosenbrock'

8.2 Interface Specification

The interface specification has the following top-level structure:

```
interface,
   <interface independent controls>
   <algebraic mappings specification>
   <simulation interface selection>
      <simulation interface dependent controls>
```

The <interface independent controls> are those controls which are valid for all interfaces. Referring to `dakota.input.summary`, these controls are defined externally from the algebraic mappings and simulation interface selection blocks (before and after). Both the algebraic mappings specification and the simulation interface selection are optional specifications, allowing the use of algebraic mappings alone, simulation-based mappings alone, or a combination. The simulation interface selection blocks are all required group specifications separated by logical OR’s, where the interface selection must be `system`, `fork`, `direct`, or `grid`. The <interface dependent controls> are those controls which are only meaningful for a specific simulation interface selection. These controls are defined within each interface selection block. Defaults for interface independent and simulation interface dependent controls are defined in `DataInterface`. The following sections provide additional detail on the interface independent controls followed by the algebraic mappings specification, the simulation interface selections, and their corresponding simulation interface dependent controls.

8.3 Interface Independent Controls

The optional set identifier specification uses the keyword `id_interface` to input a string for use in identifying a particular interface specification. A model can then identify the use of this interface by specifying the same string in its `interface_pointer` specification (see Model Commands). For example, a model whose specification contains `interface_pointer = 'I1'` will use an interface specification with `id_interface = 'I1'`. If the `id_interface` specification is omitted, a particular interface specification will be used by a model only if that model omits specifying a `interface_pointer` and if the interface set was the last set parsed (or is the only set parsed). In common practice, if only one interface set exists, then `id_interface` can be safely omitted.
from the interface specification and \texttt{interface pointer} can be omitted from the model specification(s), since there is no potential for ambiguity in this case.

Table 8.1 summarizes the set identifier interface independent control.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interface set identifier</td>
<td>\texttt{id_interface}</td>
<td>string</td>
<td>Optional</td>
<td>use of last interface parsed</td>
</tr>
</tbody>
</table>

Table 8.1: Specification detail for interface independent controls: set identifier

Table 8.2 summarizes the interface independent controls associated with parallel computing.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asynchronous interface usage</td>
<td>\texttt{asynchronous}</td>
<td>none</td>
<td>Optional</td>
<td>synchronous interface usage</td>
</tr>
<tr>
<td>Asynchronous evaluation concurrency</td>
<td>\texttt{evaluation_concurrency}</td>
<td>integer</td>
<td>Optional</td>
<td>local: unlimited concurrency, hybrid: no concurrency</td>
</tr>
<tr>
<td>Self-schedule local evals</td>
<td>\texttt{local_evaluation_self_scheduling}</td>
<td>none</td>
<td>Optional (1 of 2)</td>
<td>self-scheduled local evals</td>
</tr>
<tr>
<td>Static-schedule local evals</td>
<td>\texttt{local_evaluation_static_scheduling}</td>
<td>none</td>
<td>Optional (1 of 2)</td>
<td>self-scheduled local evals</td>
</tr>
<tr>
<td>Asynchronous analysis concurrency</td>
<td>\texttt{analysis_concurrency}</td>
<td>integer</td>
<td>Optional</td>
<td>local: unlimited concurrency, hybrid: no concurrency</td>
</tr>
<tr>
<td>Number of evaluation servers</td>
<td>\texttt{evaluation_servers}</td>
<td>integer</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
<tr>
<td>Self scheduling of evaluations</td>
<td>\texttt{evaluation_self_scheduling}</td>
<td>none</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
<tr>
<td>Static scheduling of evaluations</td>
<td>\texttt{evaluation_static_scheduling}</td>
<td>none</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
<tr>
<td>Number of analysis servers</td>
<td>\texttt{analysis_servers}</td>
<td>integer</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
<tr>
<td>Self scheduling of analyses</td>
<td>\texttt{analysis_self_scheduling}</td>
<td>none</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
<tr>
<td>Static scheduling of analyses</td>
<td>\texttt{analysis_static_scheduling}</td>
<td>none</td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
</tbody>
</table>

Table 8.2: Specification detail for interface independent controls: parallelism
The optional asynchronous flag specifies use of asynchronous protocols (i.e., background system calls, non-blocking forks, POSIX threads) when evaluations or analyses are invoked. The evaluation_concurrency and analysis_concurrency specifications serve a dual purpose:

- when running DAKOTA on a single processor in asynchronous mode, the default concurrency of evaluations and analyses is all concurrency that is available. The evaluation_concurrency and analysis_concurrency specifications can be used to limit this concurrency in order to avoid machine overload or usage policy violation.

- when running DAKOTA on multiple processors in message passing mode, the default concurrency of evaluations and analyses on each of the servers is one (i.e., the parallelism is exclusively that of the message passing). With the evaluation_concurrency and analysis_concurrency specifications, a hybrid parallelism can be selected through combination of message passing parallelism with asynchronous parallelism on each server.

The optional evaluation_servers and analysis_servers specifications support user overrides of the automatic parallel configuration for the number of evaluation servers and the number of analysis servers. Similarly, the optional evaluation_self_scheduling, evaluation_static_scheduling, analysis_self_scheduling, and analysis_static_scheduling specifications can be used to override the automatic parallel configuration of scheduling approach at the evaluation and analysis parallelism levels. That is, if the automatic configuration is undesirable for some reason, the user can enforce a desired number of partitions and a desired scheduling policy at these parallelism levels. Refer to ParallelLibrary and the Parallel Computing chapter of the Users Manual [Adams et al., 2010] for additional information.

When performing asynchronous local evaluations, the local evaluation scheduling keywords control how new evaluation jobs are dispatched when one completes. If local_evaluation_self_scheduling (default) is specified, each completed evaluation will be replaced by the next in the local evaluation queue. If local_evaluation_static_scheduling is specified, each completed evaluation will be replaced by an evaluation number congruent modulo the evaluation_concurrency. This is helpful for relative node scheduling as described in Dakota/examples/parallelism. For example, assuming only local concurrency (no MPI), if the local concurrency is 7 and job 2 completes, it will be replaced with job 9. This can result in idle processors if runtimes are non-uniform.

### 8.4 Algebraic mappings

If desired, one can define algebraic input-output mappings using the AMPL code [Fourer et al., 2003] and save these mappings in 3 files: stub.nl, stub.col, and stub.row, where stub is a particular root name describing a particular problem. These files names can be communicated to DAKOTA using the algebraic_mappings input. This string may either specify the stub.nl filename, or alternatively, just the stub itself.

DAKOTA then uses stub.col and stub.row to extract the input and output identifier strings and employs the AMPL solver library [Gay, 1997] to process the DAG specification in stub.nl. The variable and objective function names declared within AMPL should be a subset of the variable descriptors and response descriptors used by DAKOTA (see Variables Commands and Response Labels). Ordering is not important, as DAKOTA will reorder data as needed.

Table 8.3 summarizes the algebraic mappings specification.
8.5 Simulation interfaces

Each simulation interface uses one or more simulator programs, and optionally filter programs, to perform the parameter to response mapping. The simulator and filter programs are invoked with system calls, forks, direct function calls, or computational grid invocations. In the system call and fork cases, a separate process is created for the simulator program and files are used for transfer of parameter and response data between DAKOTA and the simulator program. This approach is simple and reliable and does not require any modification to simulator programs. In the direct function case, subroutine parameter lists are used to pass the parameter and response data. This approach requires modification to simulator programs so that they can be linked into DAKOTA; however it can be more efficient through the elimination of process creation overhead and deactivation of unnecessary simulator functions (e.g., output), can be less prone to loss of precision in that data can be passed directly rather than written to and read from a file, and can enable completely internal management of multiple levels of parallelism through the use of MPI communicator partitioning. In the grid case, computational grid services are utilized in order to enable distribution of simulations across different computer resources. This capability targets Condor and/or Globus services but is currently experimental and incomplete.

Table 8.4 summarizes the interface independent controls associated with the simulator programs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis drivers</td>
<td>analysis_-drivers</td>
<td>list of strings</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Additional identifiers for use by the analysis_-drivers</td>
<td>analysis_-components</td>
<td>list of strings</td>
<td>Optional</td>
<td>no additional identifiers</td>
</tr>
<tr>
<td>Input filter</td>
<td>input_filter</td>
<td>string</td>
<td>Optional</td>
<td>no input filter</td>
</tr>
<tr>
<td>Output filter</td>
<td>output_-filter</td>
<td>string</td>
<td>Optional</td>
<td>no output filter</td>
</tr>
<tr>
<td>Failure capturing</td>
<td>failure_-capture</td>
<td>abort| retry (\text{with integer data}) | recover (\text{with list of reals data}) | continuation</td>
<td>Optional group</td>
<td>abort</td>
</tr>
<tr>
<td>Feature deactivation</td>
<td>deactivate</td>
<td>active_set_-vector, evaluation_-cache, and/or restart_file</td>
<td>Optional group</td>
<td>Active set vector control, function evaluation cache, and restart file features are active</td>
</tr>
</tbody>
</table>

The required `analysis_drivers` specification provides the names of executable analysis programs or scripts.

Table 8.3: Specification detail for algebraic mappings

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic mappings file</td>
<td>algebraic_-mappings</td>
<td>string</td>
<td>Optional</td>
<td>no algebraic mappings</td>
</tr>
</tbody>
</table>

### Table 8.3: Specification detail for algebraic mappings
which comprise a function evaluation. The specification can also give values to environment variables that the programs will see; for details, see the subsection on Syntax for Filter and Driver Strings in the Interfaces chapter of the Users Manual [Adams et al., 2010]. The common case of a single analysis driver is simply accommodated by specifying a list of one driver (this also provides backward compatibility with previous DAKOTA versions). The optional `analysis_components` specification allows the user to provide additional identifiers (e.g., mesh file names) for use by the analysis drivers. This is particularly useful when the same analysis driver is to be reused multiple times for slightly different analyses. The specific content within the strings is open-ended and can involve whatever syntax is convenient for a particular analysis driver. The number of analysis components \( n_c \) should be an integer multiple of the number of drivers \( n_d \), and the first \( n_c / n_d \) component strings will be passed to the first driver, etc. The optional `input_filter` and `output_filter` specifications provide the names of separate pre- and post-processing programs or scripts which assist in mapping DAKOTA parameters files into analysis input files and mapping analysis output files into DAKOTA results files, respectively. If there is only a single analysis driver, then it is usually most convenient to combine pre- and post-processing requirements into a single analysis driver script and omit the separate input and output filters. However, in the case of multiple analysis drivers, the input and output filters provide a convenient location for non-repeated pre- and post-processing requirements. That is, input and output filters are only executed once per function evaluation, regardless of the number of analysis drivers, which makes them convenient locations for data processing operations that are shared among the analysis drivers.

Failure capturing in interfaces is governed by the optional `failure_capture` specification. Supported directives for mitigating captured failures are `abort` (the default), `retry`, `recover`, and `continuation`. The `retry` selection supports an integer input for specifying a limit on retries, and the `recover` selection supports a list of reals for specifying the dummy function values (only zeroth order information is supported) to use for the failed function evaluation. Refer to the Simulation Code Failure Capturing chapter of the Users Manual [Adams et al., 2010] for additional information.

The optional `deactivate` specification block includes three features which a user may deactivate in order to simplify interface development, increase execution speed, and/or reduce memory and disk requirements:

- **Active set vector (ASV) control**: deactivation of this feature using a `deactivate active_set_vector` specification allows the user to turn off any variability in ASV values so that active set logic can be omitted in the user's simulation interface. This option trades some efficiency for simplicity in interface development. The default behavior is to request the minimum amount of data required by an algorithm at any given time, which implies that the ASV values may vary from one function evaluation to the next. Since the user’s interface must return the data set requested by the ASV values, this interface must contain additional logic to account for any variations in ASV content. Deactivating this ASV control causes DAKOTA to always request a “full” data set (the full function, gradient, and Hessian data that is available from the interface as specified in the responses specification) on each function evaluation. For example, if ASV control has been deactivated and the responses section specifies four response functions, analytic gradients, and no Hessians, then the ASV on every function evaluation will be \( \{ 3 3 3 3 \} \), regardless of what subset of this data is currently needed. While wasteful of computations in many instances, this simplifies the interface and allows the user to return the same data set on every evaluation. Conversely, if ASV control is active (the default behavior), then the ASV requests in this example might vary from \( \{ 1 1 1 1 \} \) to \( \{ 2 0 0 2 \} \), etc., according to the specific data needed on a particular function evaluation. This will require the user’s interface to read the ASV requests and perform the appropriate logic in conditionally returning only the data requested. In general, the default ASV behavior is recommended for the sake of computational efficiency, unless interface development time is a critical concern. Note that in both cases, the data returned to DAKOTA from the user’s interface must match the ASV passed in, or else a response recovery error will result. However, when the ASV control is deactivated, the ASV values are invariant and need not be checked on every evaluation. **Note**: Deactivating the ASV control can have a positive effect on load balancing for parallel DAKOTA executions. Thus, there is significant overlap in this ASV control option with speculative
8.5 Simulation interfaces

gradients (see Method Independent Controls). There is also overlap with the mode override approach used with certain optimizers (see SNLLOptimizer and SNLLLeastSq) to combine individual value, gradient, and Hessian requests.

- Function evaluation cache: deactivation of this feature using a deactivate evaluation_cache specification allows the user to avoid retention of the complete function evaluation history in memory. This can be important for reducing memory requirements in large-scale applications (i.e., applications with a large number of variables or response functions) and for eliminating the overhead of searching for duplicates within the function evaluation cache prior to each new function evaluation (e.g., for improving speed in problems with 1000’s of inexpensive function evaluations or for eliminating overhead when performing timing studies). However, the downside is that unnecessary computations may be performed since duplication in function evaluation requests may not be detected. For this reason, this option is not recommended when function evaluations are costly. Note: duplication detection within DAKOTA can be deactivated, but duplication detection features within specific optimizers may still be active.

- Restart file: deactivation of this feature using a deactivate restart_file specification allows the user to eliminate the output of each new function evaluation to the binary restart file. This can increase speed and reduce disk storage requirements, but at the expense of a loss in the ability to recover and continue a run that terminates prematurely (e.g., due to a system crash or network problem). This option is not recommended when function evaluations are costly or prone to failure.

In addition to these simulation interface specifications, the type of interface involves a selection between system, fork, direct, or grid required group specifications. The following sections describe these group specifications in detail.

8.5.1 System call interface

Users are strongly encouraged to use the fork interface if possible, though the system interface is still supported for portability and backward compatibility. For system call interfaces, the system keyword anchors the group specification and the parameters_file, results_file, verbatim, aprepro, file_tag, and file_save are additional settings within the group specification. The parameters and results file names are supplied as strings using the parameters_file and results_file specifications. Both specifications are optional with the default data transfer files being Unix temporary files with system-generated names (e.g., /usr/tmp/aaaa08861). The parameters and results file names are passed on the command line to the analysis driver(s) and any specified input/output filters, unless the verbatim option is invoked, in which case the driver/filter invocation syntax is used verbatim without command line argument augmentation. For additional information on invocation syntax, see the Interfaces chapter of the Users Manual [Adams et al., 2010]. The format of data in the parameters files can be modified for direct usage with the APREPRO pre-processing tool [Sjaardema, 1992] using the aprepro specification (NOTE: the DPrePro pre-processing utility does not require this special formatting). File tagging (appending parameters and results files with the function evaluation number) and file saving (leaving parameters and results files in existence after their use is complete) are controlled with the file_tag and file_save flags. If these specifications are omitted, the default is no file tagging (no appended function evaluation number) and no file saving (files will be removed after a function evaluation). File tagging is most useful when multiple function evaluations are running simultaneously using files in a shared disk space, and file saving is most useful when debugging the data communication between DAKOTA and the simulation.

By default DAKOTA will remove existing results files before invoking the analysis_driver to avoid problems created by stale files in the current directory. To override this behavior and not delete existing files, specify allow_existing_files.
When performing concurrent evaluations and/or analyses, it is often necessary to cloister input and output files in separate directories to avoid conflicts. When work_directory feature is enabled, DAKOTA will create a directory for each evaluation/analysis (with optional tagging and saving as with files), and execute the analysis driver from that working directory. If an optional template_directory and/or list of template_files is specified, DAKOTA will link (or copy) those files into each working directory. The specifications for system call interfaces are summarized in Table 8.5.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>System call interface</td>
<td>system</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Parameters file name</td>
<td>parameters_file</td>
<td>string</td>
<td>Optional</td>
<td>Unix temp files</td>
</tr>
<tr>
<td>Results file name</td>
<td>results_file</td>
<td>string</td>
<td>Optional</td>
<td>Unix temp files</td>
</tr>
<tr>
<td>Allow existing results files</td>
<td>allow_existing_results</td>
<td>none</td>
<td>Optional</td>
<td>results files removed before each evaluation</td>
</tr>
<tr>
<td>Verbatim driver/filter invocation syntax</td>
<td>verbatim</td>
<td>none</td>
<td>Optional</td>
<td>driver/filter invocation syntax augmented with file names</td>
</tr>
<tr>
<td>Aprepro parameters file format</td>
<td>aprepro</td>
<td>none</td>
<td>Optional</td>
<td>standard parameters file format</td>
</tr>
<tr>
<td>Parameters and results file tagging</td>
<td>file_tag</td>
<td>none</td>
<td>Optional</td>
<td>no tagging</td>
</tr>
<tr>
<td>Parameters and results file saving</td>
<td>file_save</td>
<td>none</td>
<td>Optional</td>
<td>file cleanup</td>
</tr>
<tr>
<td>Create work directory</td>
<td>work_directory</td>
<td>none</td>
<td>Optional</td>
<td>no work directory</td>
</tr>
<tr>
<td>Name of work directory</td>
<td>named</td>
<td>string</td>
<td>Optional</td>
<td>workdir</td>
</tr>
<tr>
<td>Tag directory</td>
<td>directory_tag</td>
<td>none</td>
<td>Optional</td>
<td>no work directory tagging</td>
</tr>
<tr>
<td>Save directory</td>
<td>directory_save</td>
<td>none</td>
<td>Optional</td>
<td>remove work directory</td>
</tr>
<tr>
<td>Template directory</td>
<td>template_directory</td>
<td>string path</td>
<td>Optional</td>
<td>no template directory</td>
</tr>
<tr>
<td>Template files</td>
<td>template_files</td>
<td>list of strings</td>
<td>Optional</td>
<td>no template files</td>
</tr>
<tr>
<td>Copy template files</td>
<td>copy</td>
<td>none</td>
<td>Optional</td>
<td>link template files</td>
</tr>
<tr>
<td>Replace existing files</td>
<td>replace</td>
<td>none</td>
<td>Optional</td>
<td>do not overwrite files</td>
</tr>
</tbody>
</table>

Table 8.5: Additional specifications for system call interfaces
8.5 Simulation interfaces

8.5.2 Fork interface

For fork interfaces, the `fork` keyword anchors a group specification. The `parameters_file`, `results_file`, `verbatim`, `aprepro`, `file_tag`, and `file_save` are additional settings within this group specification and have identical meanings to those for the system call interface as summarized previously in Table 8.5.

8.5.3 Direct function interface

For direct function interfaces, `processors_per_analysis` is an additional optional setting within the required group which can be used to specify multiprocessor analysis partitions. As with the `evaluation_servers`, `analysis_servers`, `evaluation_self_scheduling`, `evaluation_static_scheduling`, `analysis_self_scheduling`, and `analysis_static_scheduling` specifications described above in Interface Independent Controls, `processors_per_analysis` provides a means for the user to override the automatic parallel configuration (refer to ParallelLibrary and the Parallel Computing chapter of the Users Manual [Adams et al., 2010]) for the number of processors used for each analysis partition. Note that if both `analysis_servers` and `processors_per_analysis` are specified and they are not in agreement, then `analysis_servers` takes precedence. The direct interface specifications are summarized in Table 8.6.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct function interface</td>
<td><code>direct</code></td>
<td><code>none</code></td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of processors per analysis</td>
<td><code>processors_per_analysis</code></td>
<td><code>integer</code></td>
<td>Optional</td>
<td>no override of auto configure</td>
</tr>
</tbody>
</table>

Table 8.6: Additional specifications for direct function interfaces

DAKOTA supports direct interfaces to a few select simulation codes. One example is ModelCenter, a commercial simulation management framework from Phoenix Integration. To utilize this interface, a user must first define the simulation specifics within a ModelCenter session and then save these definitions to a ModelCenter configuration file. The `analysis_components` specification provides the means to communicate this configuration file to DAKOTA's ModelCenter interface. A similar direct interface to The Mathworks' ([http://www.mathworks.com/](http://www.mathworks.com/)) Matlab (specified by `analysis_driver = 'matlab'`) enables a user to employ the `analysis_components` specification to point to a Matlab m-file containing a function that performs the simulation. This capability is disabled by default in DAKOTA binaries, but information on enabling and using it is available in the Users Manual [Adams et al., 2010]. Contact the DAKOTA users mailing list for assistance building and using DAKOTA with Matlab simulation support. An experimental interface to Python simulation codes is also available and can be enabled when compiling DAKOTA from source.

Other direct interfaces to simulation codes include Sandia’s SALINAS structural dynamics code, Sandia’s SIERRA multiphysics framework, and Sandia’s SAGE computational fluid dynamics code, which are available within Sandia and supported to varying degrees. In addition to interfaces with simulation codes, a common usage of the direct interface is for invoking internal test functions which are available for performing parameter to response mappings as inexpensively as possible. These problems are compiled directly into the DAKOTA executable as part of the direct function interface class and are used for algorithm testing. Refer to DirectApplication Interface for currently available testers.
8.5.4 Grid interface

For grid interfaces, no additional specifications are used at this time.

This capability has been used for interfaces with IDEA and JAVASpaces in the past and is currently a placeholder for future work with Condor and/or Globus services. It is not currently operational. The grid interface specification is summarized in Table 8.7.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid interface</td>
<td>grid</td>
<td>none</td>
<td>Required group (1 of 4 selections)</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 8.7: Additional specifications for grid interfaces
Chapter 9

Responses Commands

9.1 Responses Description

The responses specification in a DAKOTA input file specifies the data set that can be recovered from the interface after the completion of a "function evaluation." Here, the term function evaluation is used somewhat loosely to denote a data request from an iterator that is mapped through an interface in a single pass. Strictly speaking, this data request may actually involve multiple response functions and their derivatives, but the term function evaluation is widely used for this purpose. The data set is made up of a set of functions, their first derivative vectors (gradients), and their second derivative matrices (Hessians). This abstraction provides a generic data container (the Response class) whose contents are interpreted differently depending upon the type of iteration being performed. In the case of optimization, the set of functions consists of one or more objective functions, nonlinear inequality constraints, and nonlinear equality constraints. Linear constraints are not part of a response set since their coefficients can be communicated to an optimizer at start up and then computed internally for all function evaluations (see Method Independent Controls). In the case of least squares iterators, the functions consist of individual residual terms or model responses and an observed data file for comparison (as opposed to a sum of the squares objective function) as well as nonlinear inequality and equality constraints. In the case of nondeterministic iterators, the function set is made up of generic response functions for which the effect of parameter uncertainty is to be quantified. Lastly, parameter study and design of experiments iterators may be used with any of the response data set types. Within the C++ implementation, the same data structures are reused for each of these cases; only the interpretation of the data varies from iterator branch to iterator branch.

Gradient availability may be described by no_gradients, numerical_gradients, analytic_gradients, or mixed_gradients. The no_gradients selection means that gradient information is not needed in the study. The numerical_gradients selection means that gradient information is needed and will be computed with finite differences using either the native or one of the vendor finite differencing routines. The analytic_gradients selection means that gradient information is available directly from the simulation (finite differencing is not required). And the mixed_gradients selection means that some gradient information is available directly from the simulation whereas the rest will have to be estimated with finite differences.

Hessian availability may be described by no_hessians, numerical_hessians, quasi_hessians, analytic_hessians, or mixed_hessians. As for the gradient specification, the no_hessians selection indicates that Hessian information is not needed/available in the study, and the analytic_hessians selection indicates that Hessian information is available directly from the simulation. The numerical_hessians selection indicates that Hessian information is needed and will be estimated with finite differences.
using either first-order differences of gradients (for analytic gradients) or second-order differences of function values (for non-analytic gradients). The quasi_hessians specification means that Hessian information is needed and will be accumulated over time using secant updates based on the existing gradient evaluations. Finally, the mixed_hessians selection allows for a mixture of analytic, numerical, and quasi Hessian response data.

The responses specification provides a description of the total data set that is available for use by the iterator during the course of its iteration. This should be distinguished from the data subset described in an active set vector (see DAKOTA File Data Formats in the Users Manual [Adams et al., 2010]) which describes the particular subset of the response data needed for an individual function evaluation. In other words, the responses specification is a broad description of the data to be used during a study whereas the active set vector describes the particular subset of the available data that is currently needed.

Several examples follow. The first example shows an optimization data set containing an objective function and two nonlinear inequality constraints. These three functions have analytic gradient availability and no Hessian availability.

```plaintext
responses,
num_objective_functions = 1
num_nonlinear_inequality_constraints = 2
analytic_gradients
no_hessians
```

The next example shows a typical specification for a least squares data set. The six residual functions will have numerical gradients computed using the dakota finite differencing routine with central differences of 0.1% (plus/minus delta value = .001*value).

```plaintext
responses,
num_least_squares_terms = 6
numerical_gradients
    method_source dakota
    interval_type central
    fd_gradient_step_size = .001
no_hessians
```

The last example shows a specification that could be used with a nondeterministic sampling iterator. The three response functions have no gradient or Hessian availability; therefore, only function values will be used by the iterator.

```plaintext
responses,
num_response_functions = 3
no_gradients
no_hessians
```

Parameter study and design of experiments iterators are not restricted in terms of the response data sets which may be catalogued; they may be used with any of the function specification examples shown above.

### 9.2 Responses Specification

The responses specification has the following structure:

```plaintext
responses,
```
Referring to dakota.input.summary, it is evident from the enclosing brackets that the set identifier and response descriptors are optional. However, the function, gradient, and Hessian specifications are all required specifications, each of which contains several possible specifications separated by logical OR’s. The function specification must be one of three types:

- objective and constraint functions
- least squares terms and constraint functions
- generic response functions

The gradient specification must be one of four types:

- no gradients
- numerical gradients
- analytic gradients
- mixed gradients

And the Hessian specification must be one of five types:

- no Hessians
- numerical Hessians
- quasi Hessians
- analytic Hessians
- mixed Hessians

The following sections describe each of these specification components in additional detail.

### 9.3 Responses Set Identifier

The optional set identifier specification uses the keyword id_responses to input a string for use in identifying a particular responses specification. A model can then identify the use of this response set by specifying the same string in its responses_pointer specification (see Model Independent Controls). For example, a model whose specification contains responses_pointer = ‘R1’ will use a responses set with id_responses = ‘R1’.

If the id_responses specification is omitted, a particular responses specification will be used by a model only if that model omits specifying a responses_pointer and if the responses set was the last set parsed (or is the only set parsed). In common practice, if only one responses set exists, then id_responses can be safely omitted from the responses specification and responses_pointer can be omitted from the model specification(s), since there is no potential for ambiguity in this case. Table 9.1 summarizes the set identifier input.
9.4 Response Labels

The optional response labels specification uses the keyword `response_descriptors` to input a list of strings which will be replicated through the DAKOTA output to help identify the numerical values for particular response functions. The default descriptor strings use a root string plus a numeric identifier. This root string is "obj_fn" for objective functions, "least_sq_term" for least squares terms, "response_fn" for generic response functions, "nln_ineq_con" for nonlinear inequality constraints, and "nln_eq_con" for nonlinear equality constraints. Table 9.2 summarizes the response descriptors input.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response labels</td>
<td>descriptors</td>
<td>list of strings</td>
<td>Optional</td>
<td>root strings plus numeric identifiers</td>
</tr>
</tbody>
</table>

Table 9.2: Specification detail for response labels

9.5 Function Specification

The function specification must be one of three types: 1) a group containing objective and constraint functions, 2) a group containing least squares terms and constraint functions, or 3) a generic response functions specification. These function sets correspond to optimization, least squares, and uncertainty quantification iterators, respectively. Parameter study and design of experiments iterators may be used with any of the three function specifications.

9.5.1 Objective and constraint functions (optimization data set)

An optimization data set is specified using `num_objective_functions` and optionally `objective_function_scale_types`, `objective_function_scales`, `multi_objective_weights`, `num_nonlinear_inequality_constraints`, `nonlinear_inequality_lower_bounds`, `nonlinear_inequality_upper_bounds`, `nonlinear_inequality_scale_types`, `nonlinear_inequality_scales`, `num_nonlinear_equality_constraints`, `nonlinear_equality_targets`, `nonlinear_equality_scale_types`, and `nonlinear_equality_scales`. The `num_objective_functions`, `num_nonlinear_inequality_constraints`, and `num_nonlinear_equality_constraints` inputs specify the number of objective functions, nonlinear inequality constraints, and nonlinear equality constraints, respectively. The number of objective functions must be 1 or greater, and the number of inequality and equality constraints must be 0 or greater. The `objective_function_scale_types` specification includes strings specifying the scaling type for each objective function value in methods that support scaling, when scaling is enabled (see Method Independent Controls for details). Each entry in `objective_function_scale_types` may be selected from ‘none’, ‘value’, or ‘log’, to select no, characteristic value, or logarithmic scaling, respectively. Automatic scaling is not available for objective functions. If a single string is specified it will apply to each objective function. Each entry in `objective_function_scales` may be a user-specified nonzero characteristic value to be used in scaling.
each objective function. These values are ignored for scaling type ‘none’, required for ‘value’, and optional for ‘log’. If a single real value is specified it will apply to each function. If the number of objective functions is greater than 1, then a multi_objective_weights specification provides a simple weighted-sum approach to combining multiple objectives:

\[ f = \sum_{i=1}^{n} w_i f_i \]

If this is not specified, then each objective function is given equal weighting:

\[ f = \sum_{i=1}^{n} \frac{f_i}{n} \]

If scaling is specified, it is applied before multi-objective weighted sums are formed.

The nonlinear_inequality_lower_bounds and nonlinear_inequality_upper_bounds specifications provide the lower and upper bounds for 2-sided nonlinear inequalities of the form

\[ g_l \leq g(x) \leq g_u \]

The defaults for the inequality constraint bounds are selected so that one-sided inequalities of the form

\[ g(x) \leq 0.0 \]

result when there are no user constraint bounds specifications (this provides backwards compatibility with previous DAKOTA versions). In a user bounds specification, any upper bound values greater than +bigRealBoundSize (1.e+30, as defined in Minimizer) are treated as +infinity and any lower bound values less than -bigRealBoundSize are treated as -infinity. This feature is commonly used to drop one of the bounds in order to specify a 1-sided constraint (just as the default lower bounds drop out since -DBL_MAX < -big-RealBoundSize). The same approach is used for nonexistent linear inequality bounds as described in Method Independent Controls and for nonexistent design variable bounds as described in Design Variables.

The nonlinear_equality_targets specification provides the targets for nonlinear equalities of the form

\[ g(x) = g_t \]

and the defaults for the equality targets enforce a value of 0.0 for each constraint

\[ g(x) = 0.0 \]

The nonlinear_inequality_scale_types and nonlinear_equality_scale_types specifications include strings specifying the scaling type for each nonlinear inequality or equality constraint, respectively, in methods that support scaling, when scaling is enabled (see Method Independent Controls for details). Each entry in objective_function_scale_types may be selected from ‘none’, ‘value’, ‘auto’, or ‘log’, to select no, characteristic value, automatic, or logarithmic scaling, respectively. If a single string is specified it will apply to all components of the relevant nonlinear constraint vector. Each entry in nonlinear_inequality_scales and nonlinear_equality_scales may be a user-specified nonzero characteristic value to be used in scaling each constraint component. These values are ignored for scaling type ‘none’, required for ‘value’, and optional for ‘auto’ and ‘log’. If a single real value is specified it will apply to each constraint.

Any linear constraints present in an application need only be input to an optimizer at start up and do not need to be part of the data returned on every function evaluation (see the linear constraints description in Method Independent Controls). Table 9.3 summarizes the optimization data set specification.
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
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</thead>
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<tr>
<td>Number of objective functions</td>
<td>num_objective_functions</td>
<td>integer</td>
<td>Required group</td>
<td>N/A</td>
</tr>
<tr>
<td>Objective function scaling types</td>
<td>objective_function_scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
</tr>
<tr>
<td>Objective function scales</td>
<td>objective_function_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 1. (no scaling)</td>
</tr>
<tr>
<td>Multiobjective weightings</td>
<td>multi_objective_weights</td>
<td>list of reals</td>
<td>Optional</td>
<td>equal weightings</td>
</tr>
<tr>
<td>Number of nonlinear inequality constraints</td>
<td>num_nonlinear_inequality_constraints</td>
<td>integer</td>
<td>Optional</td>
<td>0</td>
</tr>
<tr>
<td>Nonlinear inequality constraint lower bounds</td>
<td>nonlinear_inequality_lower_bounds</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = -DBL_MAX</td>
</tr>
<tr>
<td>Nonlinear inequality constraint upper bounds</td>
<td>nonlinear_inequality_upper_bounds</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 0.</td>
</tr>
<tr>
<td>Nonlinear inequality constraint scaling types</td>
<td>nonlinear_inequality_scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
</tr>
<tr>
<td>Nonlinear inequality constraint scales</td>
<td>nonlinear_inequality_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 1. (no scaling)</td>
</tr>
<tr>
<td>Number of nonlinear equality constraints</td>
<td>num_nonlinear_equality_constraints</td>
<td>integer</td>
<td>Optional</td>
<td>0</td>
</tr>
<tr>
<td>Nonlinear equality constraint targets</td>
<td>nonlinear_equality_targets</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 0.</td>
</tr>
<tr>
<td>Nonlinear equality constraint scaling types</td>
<td>nonlinear_equality_scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
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<tr>
<td>Nonlinear equality constraint scales</td>
<td>nonlinear_equality_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 1. (no scaling)</td>
</tr>
</tbody>
</table>

Table 9.3: Specification detail for optimization data sets
9.5.2 Least squares terms and constraint functions (least squares data set)

A least squares data set is specified using `num_least_squares_terms` and optionally `least_squares_data_file`, `least_squares_term_scales`, `least_squares_weights`, `num_nonlinear_inequality_constraints`, `nonlinear_inequality_lower_bounds`, `nonlinear_inequality_upper_bounds`, `nonlinear_inequality_scales`, `num_nonlinear_equality_constraints`, `nonlinear_equality_targets`, and `nonlinear_equality_scales`. Each of the least squares terms is a residual function to be driven toward zero, and the nonlinear inequality and equality constraint specifications have identical meanings to those described in Objective and constraint functions (optimization data set). These types of problems are commonly encountered in parameter estimation, system identification, and model calibration. Least squares problems are most efficiently solved using special-purpose least squares solvers such as Gauss-Newton or Levenberg-Marquardt; however, they may also be solved using general-purpose optimization algorithms.

It is important to realize that, while DAKOTA can solve these problems with either least squares or optimization algorithms, the response data sets to be returned from the simulator are different. Least squares involves a set of residual functions whereas optimization involves a single objective function (sum of the squares of the residuals), i.e.,

$$ f = \sum_{i=1}^{n} (R_i)^2 $$

where $f$ is the objective function and the set of $R_i$ are the residual functions. Therefore, function values and derivative data in the least squares case involve the values and derivatives of the residual functions, whereas the optimization case involves values and derivatives of the sum of squares objective function. Switching between the two approaches will likely require different simulation interfaces capable of returning the different granularity of response data required. The specification `least_squares_data_file` may be used to specify a text file containing `num_least_squares_terms` observed data values (one per line) to be used in computing the residuals

$$ R_i = y_M - y_O $$

where $M$ denotes model and $O$, observation. In this case the simulator should return the actual model response, as DAKOTA will compute the residual internally using the supplied data.

The `least_squares_term_scale_types` specification includes strings specifying the scaling type for each least squares term in methods that support scaling, when scaling is enabled (see Method Independent Controls for details). Each entry in `least_squares_term_scale_types` may be selected from ‘none’, ‘value’, or ‘log’, to select no, characteristic value, or logarithmic scaling, respectively. Automatic scaling is not available for least squares terms. If a single string is specified it will apply to each least squares terms. Each entry in `least_squares_term_scales` may be a user-specified nonzero characteristic value to be used in scaling each term. These values are ignored for scaling type ‘none’, required for ‘value’, and optional for ‘log’. If a single real value is specified it will apply to each term. The `least_squares_weights` specification provides a means to multiplicatively weight the vector of least squares residuals with a vector of weights. If scaling is specified, it is applied before term weighting.

Table 9.4 summarizes the least squares data set specification.

9.5.3 Response functions (generic data set)

A generic response data set is specified using `num_response_functions`. Each of these functions is simply a response quantity of interest with no special interpretation taken by the method in use. This type of data set is used by uncertainty quantification methods, in which the effect of parameter uncertainty on response functions is
<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of least squares terms</td>
<td>num_least_squares_terms</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
<tr>
<td>Least squares data source file</td>
<td>least_squares_data_file</td>
<td>string</td>
<td>Optional</td>
<td>none</td>
</tr>
<tr>
<td>Least squares term scaling types</td>
<td>least_squares_term_scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
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<td>Least squares terms scales</td>
<td>least_squares_term_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>no scaling (vector values = 1.)</td>
</tr>
<tr>
<td>Least squares terms weightings</td>
<td>least_squares_weights</td>
<td>list of reals</td>
<td>Optional</td>
<td>equal weightings</td>
</tr>
<tr>
<td>Number of nonlinear inequality constraints</td>
<td>num_nonlinear_inequality_constraints</td>
<td>integer</td>
<td>Optional</td>
<td>0</td>
</tr>
<tr>
<td>Nonlinear inequality constraint lower bounds</td>
<td>nonlinear_inequality_lower_bounds</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = -DBL_MAX</td>
</tr>
<tr>
<td>Nonlinear inequality constraint upper bounds</td>
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<td>list of reals</td>
<td>Optional</td>
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<tr>
<td>Nonlinear inequality scaling types</td>
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<td>Optional</td>
<td>vector values = 'none'</td>
</tr>
<tr>
<td>Nonlinear inequality constraint scales</td>
<td>nonlinear_inequality_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>no scaling (vector values = 1.)</td>
</tr>
<tr>
<td>Number of nonlinear equality constraints</td>
<td>num_nonlinear_equality_constraints</td>
<td>integer</td>
<td>Optional</td>
<td>0</td>
</tr>
<tr>
<td>Nonlinear equality constraint targets</td>
<td>nonlinear_equality_targets</td>
<td>list of reals</td>
<td>Optional</td>
<td>vector values = 0.</td>
</tr>
<tr>
<td>Nonlinear equality scaling types</td>
<td>nonlinear_equality_scale_types</td>
<td>list of strings</td>
<td>Optional</td>
<td>vector values = 'none'</td>
</tr>
<tr>
<td>Nonlinear equality constraint scales</td>
<td>nonlinear_equality_scales</td>
<td>list of reals</td>
<td>Optional</td>
<td>no scaling (vector values = 1.)</td>
</tr>
</tbody>
</table>

Table 9.4: Specification detail for nonlinear least squares data sets
quantified, and can also be used in parameter study and design of experiments methods (although these methods are not restricted to this data set), in which the effect of parameter variations on response functions is evaluated. Whereas objective, constraint, and residual functions have special meanings for optimization and least squares algorithms, the generic response function data set need not have a specific interpretation and the user is free to define whatever functional form is convenient. Table 9.5 summarizes the generic response function data set specification.

<table>
<thead>
<tr>
<th>Description</th>
<th>Keyword</th>
<th>Associated Data</th>
<th>Status</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of response functions</td>
<td>num_response_functions</td>
<td>integer</td>
<td>Required</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 9.5: Specification detail for generic response function data sets

### 9.6 Gradient Specification

The gradient specification must be one of four types: 1) no gradients, 2) numerical gradients, 3) analytic gradients, or 4) mixed gradients.

#### 9.6.1 No gradients

The no_gradients specification means that gradient information is not needed in the study. Therefore, it will neither be retrieved from the simulation nor computed with finite differences. The no_gradients keyword is a complete specification for this case.

#### 9.6.2 Numerical gradients

The numerical_gradients specification means that gradient information is needed and will be computed with finite differences using either the native or one of the vendor finite differencing routines.

The method_source setting specifies the source of the finite differencing routine that will be used to compute the numerical gradients: dakota denotes DAKOTA’s internal finite differencing algorithm and vendor denotes the finite differencing algorithm supplied by the iterator package in use (DOT, CONMIN, NPSOL, NL2SOL, NLSSOL, and OPT++ each have their own internal finite differencing routines). The dakota routine is the default since it can execute in parallel and exploit the concurrency in finite difference evaluations (see Exploiting Parallelism in the Users Manual [Adams et al., 2010]). However, the vendor setting can be desirable in some cases since certain libraries will modify their algorithm when the finite differencing is performed internally. Since the selection of the dakota routine hides the use of finite differencing from the optimizers (the optimizers are configured to accept user-supplied gradients, which some algorithms assume to be of analytic accuracy), the potential exists for the vendor setting to trigger the use of an algorithm more optimized for the higher expense and/or lower accuracy of finite-differencing. For example, NPSOL uses gradients in its line search when in user-supplied gradient mode (since it assumes they are inexpensive), but uses a value-based line search procedure when internally finite differencing. The use of a value-based line search will often reduce total expense in serial operations. However, in parallel operations, the use of gradients in the NPSOL line search (user-supplied gradient mode) provides excellent load balancing without need to resort to speculative optimization approaches. In summary, then, the dakota routine is preferred for parallel optimization, and the vendor routine may be preferred for serial optimization in special cases.
The interval_type setting is used to select between forward and central differences in the numerical gradient calculations. The dakota, DOT vendor, and OPT++ vendor routines have both forward and central differences available, the CONMIN and NL2SOL vendor routines support forward differences only, and the NPSOL and NLSSOL vendor routines start with forward differences and automatically switch to central differences as the iteration progresses (the user has no control over this). The following forward difference expression

$$\nabla f(x) \approx \frac{f(x + he_i) - f(x)}{h}$$

and the following central difference expression

$$\nabla f(x) \approx \frac{f(x + he_i) - f(x - he_i)}{2h}$$

are used to estimate the \(i^{th}\) component of the gradient vector.

Lastly, fd_gradient_step_size specifies the relative finite difference step size to be used in the computations. Either a single value may be entered for use with all parameters, or a list of step sizes may be entered, one for each parameter. The latter option of a list of step sizes is only valid for use with the DAKOTA finite differencing routine. For DAKOTA, DOT, CONMIN, and OPT++, the differencing intervals are computed by multiplying the fd_gradient_step_size with the current parameter value. In this case, a minimum absolute differencing interval is needed when the current parameter value is close to zero. This prevents finite difference intervals for the parameter which are too small to distinguish differences in the response quantities being computed. DAKOTA, DOT, CONMIN, and OPT++ will use intervals of \(.01\times\text{current value with a minimum interval of 1.e-5.}\) NPSOL and NLSSOL use a different formula for their finite difference intervals: \(\text{fd_gradient_step_size} \times (1+|\text{current parameter value}|).\) This definition has the advantage of eliminating the need for a minimum absolute differencing interval since the interval no longer goes to zero as the current parameter value goes to zero.

When DAKOTA computes gradients or Hessians by finite differences and the variables in question have bounds, it by default chooses finite-differencing steps that keep the variables within their specified bounds. Older versions of DAKOTA generally ignored bounds when computing finite differences. To restore the older behavior, one can add keyword ignore_bounds to the response specification when method_source dakota (or just dakota) is also specified. In forward difference or backward difference computations, honoring bounds is straightforward. To honor bounds when approximating \(\frac{\partial f}{\partial x_i}, i.e.,\) component \(i\) of the gradient of \(f\), by central differences, DAKOTA chooses two steps \(h_1\) and \(h_2\) with \(h_1 \neq h_2\), such that \(x + h_1e_i\) and \(x + h_2e_i\) both satisfy the bounds, and then computes

$$\frac{\partial f}{\partial x_i} \approx \frac{h_2^2(f_1 - f_0) - h_1^2(f_2 - f_0)}{h_1h_2(h_2 - h_1)}$$

with \(f_0 = f(x), f_1 = f(x + h_1e_i),\) and \(f_2 = f(x + h_2e_i).\)

Table 9.6 summarizes the numerical gradient specification.

9.6.3 Analytic gradients

The analytic_gradients specification means that gradient information is available directly from the simulation (finite differencing is not required). The simulation must return the gradient data in the DAKOTA format (enclosed in single brackets; see DAKOTA File Data Formats in the Users Manual [Adams et al., 2010]) for the case of file transfer of data. The analytic_gradients keyword is a complete specification for this case.
9.6 Gradient Specification

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<thead>
<tr>
<th>Description</th>
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<td>dakota</td>
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<td>Optional group</td>
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<td>Interval type</td>
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<td>forward</td>
<td>central</td>
<td>Optional group</td>
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<td>bounds respected</td>
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Table 9.6: Specification detail for numerical gradients

9.6.4 Mixed gradients

The mixed_gradients specification means that some gradient information is available directly from the simulation (analytic) whereas the rest will have to be finite differenced (numerical). This specification allows the user to make use of as much analytic gradient information as is available and then finite difference for the rest. For example, the objective function may be a simple analytic function of the design variables (e.g., weight) whereas the constraints are nonlinear implicit functions of complex analyses (e.g., maximum stress). The id_analytic_gradients list specifies by number the functions which have analytic gradients, and the id_numerical_gradients list specifies by number the functions which must use numerical gradients. Each function identifier, from 1 through the total number of functions, must appear once and only once within the union of the id_analytic_gradients and id_numerical_gradients lists. The method_source, interval_type, and fd_gradient_step_size specifications are as described previously in Numerical gradients and pertain to those functions listed by the id_numerical_gradients list. Table 9.7 summarizes the mixed gradient specification.

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<td>Interval type</td>
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<td>forward</td>
<td>central</td>
<td>Optional group</td>
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Table 9.7: Specification detail for mixed gradients
9.7 Hessian Specification

Hessian availability must be specified with either no_hessians, numerical_hessians, quasi_hessians, analytic_hessians, or mixed_hessians.

9.7.1 No Hessians

The no_hessians specification means that the method does not require DAKOTA to manage the computation of any Hessian information. Therefore, it will neither be retrieved from the simulation nor computed by DAKOTA. The no_hessians keyword is a complete specification for this case. Note that, in some cases, Hessian information may still be being approximated internal to an algorithm (e.g., within a quasi-Newton optimizer such as optpp_q_newton); however, DAKOTA has no direct involvement in this process and the responses specification need not include it.

9.7.2 Numerical Hessians

The numerical_hessians specification means that Hessian information is needed and will be computed with finite differences using either first-order gradient differencing (for the cases of analytic_gradients or for the functions identified by id_analytic_gradients in the case of mixed_gradients) or first- or second-order function value differencing (all other gradient specifications). In the former case, the following expression

$$\nabla^2 f(x)_i \approx \frac{\nabla f(x + he_i) - \nabla f(x)}{h}$$

estimates the $i^{th}$ Hessian column, and in the latter case, the following expressions

$$\nabla^2 f(x)_{i,j} \approx \frac{f(x + he_i + he_j) - f(x + he_i) - f(x - he_i + he_j) + f(x - he_i) + f(x - he_i)}{4h^2}$$

and

$$\nabla^2 f(x)_{i,j} \approx \frac{f(x + he_i + he_j) - f(x + he_i - he_j) - f(x - he_i + he_j) + f(x - he_i - he_j)}{4h^2}$$

provide first- and second-order estimates of the $ij^{th}$ Hessian term. Prior to DAKOTA 5.0, DAKOTA always used second-order estimates. In DAKOTA 5.0 and newer, the default is to use first-order estimates (which honor bounds on the variables and require only about a quarter as many function evaluations as do the second-order estimates), but specifying central after numerical_hessians causes DAKOTA to use the old second-order estimates, which do not honor bounds. In optimization algorithms that use Hessians, there is little reason to use second-order differences in computing Hessian approximations.

The fd_hessian_step_size specifies the relative finite difference step size to be used in these differences. Either a single value may be entered for use with all parameters, or a list of step sizes may be entered, one for each parameter. The differencing intervals are computed by multiplying the fd_hessian_step_size with the current parameter value. A minimum absolute differencing interval of $0.01 \cdot \text{fd}_\text{hessian}_\text{step}_\text{size}$ is used when the current parameter value is close to zero. Table 9.8 summarizes the numerical Hessian specification.

9.7.3 Quasi Hessians

The quasi_hessians specification means that Hessian information is needed and will be approximated using secant updates (sometimes called “quasi-Newton updates”, though any algorithm that approximates Newton’s
### 9.7 Hessian Specification

<table>
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Table 9.8: Specification detail for numerical Hessians

Method is a quasi-Newton method. Compared to finite difference numerical Hessians, secant approximations do not expend additional function evaluations in estimating all of the second-order information for every point of interest. Rather, they accumulate approximate curvature information over time using the existing gradient evaluations. The supported secant approximations include the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update (specified with the keyword `bfgs`)

\[
B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k}
\]

and the Symmetric Rank 1 (SR1) update (specified with the keyword `sr1`)

\[
B_{k+1} = B_k + \frac{(y_k - B_k s_k)(y_k - B_k s_k)^T}{(y_k - B_k s_k)^T s_k}
\]

where \( B_k \) is the \( k \)th approximation to the Hessian, \( s_k = x_{k+1} - x_k \) is the step and \( y_k = \nabla f_{k+1} - \nabla f_k \) is the corresponding yield in the gradients. In both cases, an initial scaling of \( y_k^T s_k I \) is used for \( B_0 \) prior to the first update. In addition, both cases employ basic numerical safeguarding to protect against numerically small denominators within the updates. This safeguarding skips the update if \( |y_k^T s_k| < 10^{-6} \) in the BFGS case or if \( |(y_k - B_k s_k)^T s_k| < 10^{-6} \) in the SR1 case. In the BFGS case, additional safeguarding can be added using the `damped` option, which utilizes an alternative damped BFGS update when the curvature condition \( y_k^T s_k > 0 \) is nearly violated. Table 9.9 summarizes the quasi Hessian specification.

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Table 9.9: Specification detail for quasi Hessians

#### 9.7.4 Analytic Hessians

The `analytic_hessians` specification means that Hessian information is available directly from the simulation. The simulation must return the Hessian data in the DAKOTA format (enclosed in double brackets; see DAKOTA File Data Formats in Users Manual [Adams et al., 2010]) for the case of file transfer of data. The `analytic_hessians` keyword is a complete specification for this case.
9.7.5 Mixed Hessians

The `mixed_hessians` specification means that some Hessian information is available directly from the simulation (analytic) whereas the rest will have to be estimated by finite differences (numerical) or approximated by secant updating. As for mixed gradients, this specification allows the user to make use of as much analytic information as is available and then estimate/approximate the rest. The `id_analytic_hessians` list specifies by number the functions which have analytic Hessians, and the `id_numerical_hessians` and `id_quasi_hessians` lists specify by number the functions which must use numerical Hessians and secant Hessian updates, respectively. Each function identifier, from 1 through the total number of functions, must appear once and only once within the union of the `id_analytic_hessians`, `id_numerical_hessians`, and `id_quasi_hessians` lists. The `fd_hessian_step_size` and `bfgs`, `damped bfgs`, or `sr1` secant update selections are as described previously in Numerical Hessians and Quasi Hessians and pertain to those functions listed by the `id_numerical_hessians` and `id_quasi_hessians` lists. Table 9.10 summarizes the mixed Hessian specification.

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Table 9.10: Specification detail for mixed Hessians
Chapter 10

Bibliography


